1 Functions f and g are defined on the set of real numbers by

- $f(x) = x^2 + 3$
- $g(x) = x + 4$

(a) Find expressions for:

(i) $f(g(x))$;

(ii) $g(f(x))$. **3**

Generic Scheme			Illustrative Scheme	
1 (a)				
•1	ic	start composite process	•1 e.g. $f(x+4)$	**stated, or implied by •2**
•2	ic	correct substitution into expression	•2 $(x+4)^2+3$	
•3	ic	complete second composite	•3 x^2+3+4	3

Notes

1. Candidates must clearly identify which of their answers are $f(g(x))$ and $g(f(x))$; the minimum evidence for this could be as little as using (i) and (ii) as labels.
2. Candidates who interpret the composite functions as either $f(x)\times g(x)$ or $f(x)+g(x)$, do not gain any marks.

Regularly occurring responses

Response 1 : The first two marks are for **either** $f(g(x))$ **or** $g(f(x))$ correct. The third mark is for the other composite function.

Candidate A	**Candidate B**
$f(g(x))$	$f(g(x))$
$=(x+4)^2+3$ ✓ •1 ✓ •2	$=(x+7)^2$ ✗ •3
$g(f(x))$	$g(f(x))$
$=x^2+12$ ✗ •3	$=x^2+7$ ✓ •1 ✓ •2
2 marks out of 3	2 marks out of 3

Response 2 : Interpreting $f(g(x))$ as $g(f(x))$ and vice versa. A maximum of 2 marks are available.

Candidate C	**Candidate D**
$f(g(x))$	$f(g(x))$
$=x^2+7$ ✗ •1 ✗̸ •2	$=x^2+7$ ✗ •1 ✗̸ •2
$g(f(x))$	
$=(x+4)^2+3$ ✗̸ •3	
2 marks out of 3	1 mark out of 3

Response 3 : Identifying $f(g(x))$ and $g(f(x))$

Candidate E	**Candidate F**	**Candidate G**	**Candidate H**
$(x+4)^2+3$ ✗ •1 ✓ •2	x^2+7 ✗ •1 ✗ •2	x^2+7 ONLY	(i) $(x+4)^2+3$ ✓ •1 ✓ •2
x^2+7 ✓ •3	$(x+4)^2+3$ ✗̸ •3	**or** $(x+4)^2+3$ ONLY	(ii) x^2+7 ✓ •3
2 marks out of 3	1 mark out of 3	0 marks out of 3	3 marks out of 3

1	(b)	Show that $f(g(x)) + g(f(x)) = 0$ has no real roots.	3

Generic Scheme | **Illustrative Scheme**

1 (b)

Generic Scheme	Illustrative Scheme
Method 1 : Discriminant	Method 1 : Discriminant
•4 pd obtain a quadratic expression	•4 $2x^2 + 8x + 26$
•5 ss know to and use discriminant	•5 $8^2 - 4 \times 2 \times 26$ or $4^2 - 4 \times 1 \times 13$ **stated, or implied by •6**
•6 ic interpret result	•6 $-144 < 0$ or $-36 < 0$ so no real roots
Method 2 : Quadratic Formula	Method 2 : Quadratic Formula
•4 pd obtain a quadratic expression	•4 $2x^2 + 8x + 26$
•5 ss know to and use quadratic formula	•5 $\dfrac{-8 \pm \sqrt{8^2 - 4 \times 2 \times 26}}{2 \times 2}$ **stated, or implied by •6**
•6 ic interpret result	•6 $\sqrt{-144}$ not possible so no real roots

3

Notes

3. Candidates who use $f(x) \times g(x)$ can gain no marks in (b) as a cubic will be obtained.
4. Candidates who use $f(x) + g(x)$ do not gain •4 (eased) but •5 and •6 are available as follow through marks.
5. In method 1, any other formula masquerading as a discriminant cannot gain •5 and •6.
6. •4, •5 and •6 are only available if $f(g(x)) + g(f(x))$ simplifies to a quadratic expression of the form $ax^2 + bx + c$, with b and c both non-zero.
7. •6 is only available for a numerical value, calculated correctly from the candidate's response at •4, and leading to no real roots.
8. Do not accept for •6:
   - 'no roots' in lieu of 'no real roots'
   - 'maths error' or 'ma error'.
9. Candidates who use the word derivative instead of discriminant should not be penalised.

Regularly occurring responses

Response 4 : Candidates who do not simplify the value of their discriminant

Candidate I

$8^2 - 4 \times 2 \times 26$ ✓ •5 ✓

$= 64 - 208 < 0$ so no real roots •6 ✗

Response 5 : Acceptable communication marks

Method 1

Candidate J

$\sqrt{8^2 - 4 \times 2 \times 26}$ ✓ •5

$= \sqrt{-144}$

not valid

so no real roots ✓ •6

Candidate K

Discriminant $= \sqrt{8^2 - 4 \times 2 \times 26}$ ✓ •5

$= \sqrt{-144}$

can't find root of negative

so no real roots ✓ •6

Candidate L

no real roots if $b^2 - 4ac < 0$

$64 - 208 = -144$ ✓ •6

✓ •5

Method 2

Candidate M

$\dfrac{-(-4) \pm \sqrt{8^2 - 4 \times 2 \times 26}}{2 \times 2}$ ✓ •5

$= \dfrac{4 \pm \sqrt{-144}}{4}$

no $\sqrt{-\text{ve}}$

so no real roots ✓ •6

2 (a) Relative to a suitable set of coordinate axes, diagram 1 shows the line $2x - y + 5 = 0$ intersecting the circle $x^2 + y^2 - 6x - 2y - 30 = 0$ at the points P and Q.

Find the coordinates of P and Q. **6**

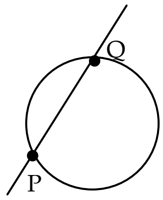


Diagram 1

Generic Scheme	**Illustrative Scheme**
2 (a)	
	Substituting for y
$\bullet^1$ ss rearrange linear equation	$\bullet^1$ $y = 2x + 5$ **stated, or implied by** $\bullet^2$
$\bullet^2$ ss substitute into circle	$\bullet^2$ $\ldots(2x+5)^2 \ldots - 2(2x+5)\ldots$
$\bullet^3$ pd express in standard form	$\bullet^3$ $5x^2 + 10x - 15$ } $= 0$ must appear at the $\bullet^3$ or $\bullet^4$ stage to gain $\bullet^3$.
$\bullet^4$ pd start to solve	$\bullet^4$ e.g. $5(x+3)(x-1)$
$\bullet^5$ ic state roots	$\bullet^5$ $x = -3$ and $x = 1$
$\bullet^6$ pd determine corresponding y-coordinates	$\bullet^6$ $y = -1$ and $y = 7$
	Substituting for x
	$\bullet^1$ $x = \dfrac{y-5}{2}$ **stated, or implied by** $\bullet^2$
	$\bullet^2$ $\left(\dfrac{y-5}{2}\right)^2 \ldots - 6\left(\dfrac{y-5}{2}\right)\ldots$
	$\bullet^3$ $5y^2 - 30y - 35$ } $= 0$ must appear at the $\bullet^3$ or $\bullet^4$ stage to gain $\bullet^3$.
	$\bullet^4$ e.g. $5(y+1)(y-7)$
	$\bullet^5$ $y = -1$ and $y = 7$
	$\bullet^6$ $x = -3$ and $x = 1$
	6

Notes

1. At $\bullet^4$ the quadratic must lead to two real distinct roots for $\bullet^5$ and $\bullet^6$ to be available.
2. Cross marking is available here for $\bullet^5$ and $\bullet^6$.
3. Candidates do not need to distinguish between points P and Q.

Regularly occurring responses

Response 1 : Solving quadratic equation

Candidate A

✓ $\bullet^1$ ✓ $\bullet^2$

$5x^2 + 10x + 5 = 0$ ✗ $\bullet^3$

$5(x+1)(x+1)$ ✗ $\bullet^4$

$x = -1$ ✗ $\bullet^5$

$y = 3$ ✗ $\bullet^6$

Cross marking is **not** available here for $\bullet^5$ and $\bullet^6$, as there are no distinct roots. See Note 1.

Candidate B

$y = 2x + 5$ ✓ $\bullet^1$

$x^2 + (2x+5)^2 - 6x - 2(7x+5) - 30 = 0$ ✗ $\bullet^2$

$5x^2 - 15 = 0$ ✗ $\bullet^3$

$x^2 = 3$ ✗ $\bullet^4$

$x = \pm\sqrt{3}$ ✗ $\bullet^5$

$y = 8\cdot5,\ 1\cdot5$ ✗ $\bullet^6$

Candidate C

✓ $\bullet^1$ ✓ $\bullet^2$

$5x^2 + 10x - 15 = 0$ ✓ $\bullet^3$

$5x^2 + 10x = 15$

$5x(x+2) = 15$ ✗ $\bullet^4$

$x(x+2) = 3$

$x = 3 \quad x = 1$ ✗ $\bullet^5$

$y = 11 \quad y = 7$ ✗ $\bullet^6$

2 (b) Diagram 2 shows the circle from (a) and a second congruent circle, which also passes through P and Q.

Determine the equation of this second circle.

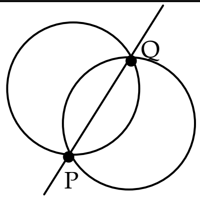


Diagram 2

6

Generic Scheme			Illustrative Scheme
2 (b)			
$\bullet^7$	ic	centre of original circle	$\bullet^7$ (3, 1)
$\bullet^8$	pd	radius of original circle	$\bullet^8$ $\sqrt{40}$ Accept $r^2 = 40$
Method 1 : Using midpoint			Method 1 : Using midpoint
$\bullet^9$	ss	midpoint of chord	$\bullet^9$ (–1, 3)
$\bullet^{10}$	ss	evidence for finding new centre	$\bullet^{10}$ e.g. stepping out or midpoint formula
$\bullet^{11}$	ic	centre of new circle	$\bullet^{11}$ (–5, 5)
$\bullet^{12}$	ic	equation of new circle	$\bullet^{12}$ $(x+5)^2+(y-5)^2=40$
Method 2 : Stepping out using P and Q			Method 2 : Stepping out using P and Q
$\bullet^9$	ss	evidence of C_1 to P **or** C_1 to Q	$\bullet^9$ e.g. stepping out or vector approach
$\bullet^{10}$	ss	evidence of Q to C_2 **or** P to C_2	$\bullet^{10}$ e.g. stepping out or vector approach
$\bullet^{11}$	ic	centre of new circle	$\bullet^{11}$ (–5, 5)
$\bullet^{12}$	ic	equation of new circle	$\bullet^{12}$ $(x+5)^2+(y-5)^2=40$ **6**

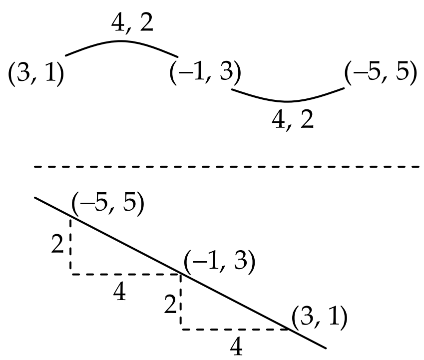
Notes

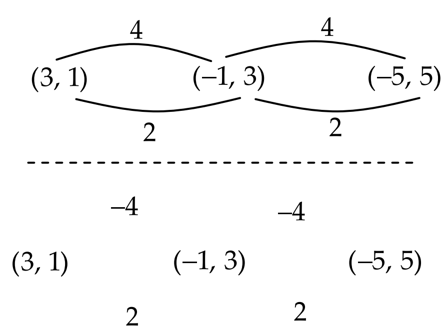
4. The evidence for $\bullet^7$ and $\bullet^8$ may appear in (a).
5. Centre (–5, 5) **without working** in method 1 may still gain $\bullet^{12}$ but not $\bullet^{10}$ or $\bullet^{11}$, in method 2 may still gain $\bullet^{12}$ but not $\bullet^9$, $\bullet^{10}$ or $\bullet^{11}$.

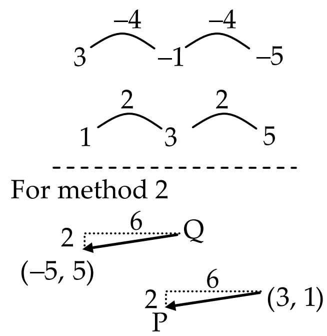
   Any other centre **without working** in method 1 does not gain $\bullet^{10}$, $\bullet^{11}$ or $\bullet^{12}$, in method 2 does not gain $\bullet^9$, $\bullet^{10}$, $\bullet^{11}$ or $\bullet^{12}$.
6. The centre must have been clearly indicated before it is used at the $\bullet^{12}$ stage.
7. Do not accept e.g. $\sqrt{40}^2$ or $39\cdot69$, or any other decimal approximations for $\bullet^{12}$.
8. The evidence for $\bullet^8$ may not appear until the candidate states the radius or equation of the second circle.

Regularly occurring responses

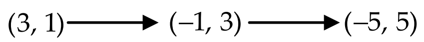
Response 2 : Examples of evidence for stepping out for $\bullet^{10}$ in method 1 or $\bullet^9$ or $\bullet^{10}$ in method 2

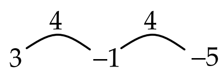


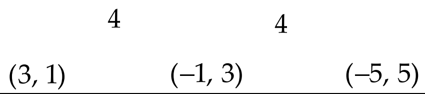




Response 3 : Examples of evidence which do not gain $\bullet^{10}$ in method 1 for stepping out







3	A function f is defined on the domain $0 \le x \le 3$ by $f(x) = x^3 - 2x^2 - 4x + 6$.	
	Determine the maximum and minimum values of f.	7

Generic Scheme | **Illustrative Scheme**

3

	Generic Scheme		Illustrative Scheme
$\bullet^1$	ss start to differentiate	$\bullet^1$	differentiate x^3 or $-2x^2$ correctly
$\bullet^2$	ss complete derivative and set to 0	$\bullet^2$	$3x^2 - 4x - 4$
$\bullet^3$	pd start to solve $f'(x) = 0$	$\bullet^3$	e.g. $(3x+2)(x-2)$
$\bullet^4$	pd solve $f'(x) = 0$	$\bullet^4$	$-\frac{2}{3}$, 2
$\bullet^5$	ic evaluate f at relevant stationary point	$\bullet^5$	$f(2) = -2$
$\bullet^6$	ss consider end-points	$\bullet^6$	$f(0) = 6$ **and** $f(3) = 3$
$\bullet^7$	ic state max. and min. values	$\bullet^7$	max. 6 **and** min. -2

($\bullet^2$ and $\bullet^3$: $= 0$ must appear at $\bullet^2$ or $\bullet^3$ to gain $\bullet^2$.)

7

Notes

1. The only valid approach is via differentiation. A numerical approach can only gain $\bullet^6$.
2. Candidates who consider stationary points only cannot gain $\bullet^6$ or $\bullet^7$.
3. Treat maximum $(0, 6)$ and minimum $(2, -2)$ as bad form.
4. Cross marking is **not** applicable to $\bullet^6$ or $\bullet^7$.
5. Ignore any nature table which may appear in a candidate's solution, however $(2, -2)$ at table is sufficient for $\bullet^5$.

Regularly occurring responses

Response 1 : Algebraic issues in working

Candidate A

Working	
$y' = 3x^2 - 4x - 4$	$\bullet^1$ ✓
$(3x-2)(x+2)$ ✗	$\bullet^2$ ✗
$x = \frac{2}{3}$, $x = -2$ ✓	$\bullet^3$ ✗
When $x = \frac{2}{3}$, $y = \frac{74}{27}$ ✓	$\bullet^4$ ✓
$f(0) = 6$ and $f(3) = 3$ ✓	$\bullet^5$ ✓
max $= 6$, min $= 2\frac{20}{27}$ ✓	$\bullet^6$ ✓
	$\bullet^7$ ✓

Candidate B

Working	
$3x^2 - 4x - 4 = 0$ ✓	$\bullet^1$ ✓
$(3x-2)(x-2)$ ✗	$\bullet^2$ ✓
$x = \frac{2}{3}$ or $x = 2$ ✓	$\bullet^3$ ✗
so $f(2) = -2$ ✗	$\bullet^4$ ✓
	$\bullet^5$ ✗

Since $\frac{2}{3}$ is within the domain, $f\left(\frac{2}{3}\right)$ must also be calculated to gain $\bullet^5$.

Candidate C

Working	
$3x^2 - 4x - 4$	$\bullet^1$ ✓
$(3x+2)(x-2)$ ✓	$\bullet^2$ ✗
$3x+2=0$ $x-2=0$	$\bullet^3$ ✓
$x = -\frac{2}{3}$ $x = 2$ ✓	$\bullet^4$ ✓
$f(2) = -2$ ✓	$\bullet^5$ ✓

Ignore the value of $f\left(-\frac{2}{3}\right)$ here, if it is included.

Response 2 : Derivative not explicitly set to zero

Candidate D

$f'(x) = 3x^2 - 4x - 4$ ✓ $\bullet^1$
$f'(x) = 0$ ✓ $\bullet^2$

Candidate E

$f'(x) = 0$
$f'(x) = 3x^2 - 4x - 4$ ✓ $\bullet^1$ ✓ $\bullet^2$
$= (3x+2)(x-2)$ ✓ $\bullet^3$

Candidate F

$f'(x) = 0$
$3x^2 - 4x - 4$ ✓ $\bullet^1$ ✗ $\bullet^2$
$= (3x+2)(x-2)$ ✓ $\bullet^3$

Candidate G

$f'(x) = 0$ only ✗ $\bullet^1$ ✗ $\bullet^2$

3 continued

Regularly occurring responses

Response 3 : Solving quadratic equation

Candidate H

$f'(x) = 3x^2 - 4x - 4$ ✓ $\bullet^1$ ✓

$3x^2 - 4x - 4 = 0$ ✓ $\bullet^2$ ✓

$3x^2 - 4x = 4$ $\bullet^3$ ✗

$x(3x - 4) = 4$ $\bullet^4$ ✗

$x = 4, \frac{4}{3}$ $\bullet^5$ ✗

Due to 'method' chosen $\bullet^3$, $\bullet^4$, $\bullet^5$ and $\bullet^7$ are not available.

Candidate I

✓ $\bullet^1$

$3x^2 - 4x - 4 = 0$ ✓ $\bullet^2$

$x = \dfrac{-(-4) \pm \sqrt{(4)^2 - 4 \times 3 \times (-4)}}{2 \times 3}$ ✓ $\bullet^3$

Ignore omission of negative sign at square here.

Response 4 : Numerical approach

Candidate J

$f(0) = 6$

$f(3) = 3$ ✓ $\bullet^6$

This candidate has stayed within the interval $0 \le x \le 3$.

Candidate K

$f(0) = 6$

$f(1) = 1$

$f(2) = -2$ ✗ $\bullet^5$

$f(3) = 3$ ✓ $\bullet^6$

Candidate L

$f(0) = 6$

$f(1) = 1$

$f(2) = -2$ ✗ $\bullet^5$

$f(3) = 3$ ✗ $\bullet^6$

$f(4) = 22$

This candidate has gone outwith the interval $0 \le x \le 3$.

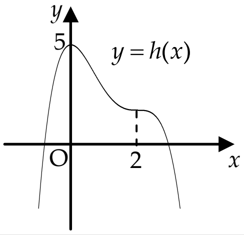
For $\bullet^5$, $f(2)$ must come from calculus and not from any other approach.

4 The diagram below shows the graph of a quartic $y = h(x)$, with stationary points at $x = 0$ and $x = 2$.

On separate diagrams sketch the graphs of:

(a) $y = h'(x)$; **3**

(b) $y = 2 - h'(x)$. **3**



Generic Scheme			Illustrative Scheme
4 (a)			
•[1]	ic	identify roots	•[1] 0 **and** 2 only
•[2]	ic	interpret point of inflection	•[2] turning point at (2, 0)
•[3]	ic	complete cubic curve	•[3] cubic, passing through O with negative gradient
			3

Notes

1. All graphs must include both the x and y axes (labelled or unlabelled), however the origin need not be labelled.
2. No marks are available unless a graph is attempted.
3. No marks are available to a candidate who makes several attempts at a graph on the same diagram, unless it is clear which is the final graph.
4. A linear graph gains no marks in both (a) and (b).

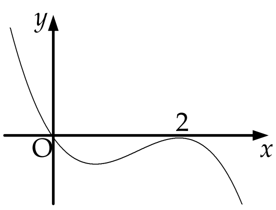
Generic Scheme			Illustrative Scheme
4 (b)			
•[4]	ic	reflection in x-axis	•[4] reflection of graph in (a) in x-axis
•[5]	ic	translation $\begin{bmatrix} 0 \\ 2 \end{bmatrix}$	•[5] graph moves parallel to y-axis by 2 units upwards
•[6]	ic	annotation of 'transformed' graph	•[6] two 'transformed' points appropriately annotated (see Note 5)
			3

Notes

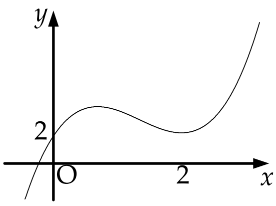
5. 'Transformed' here means a reflection followed by a translation.
6. •[4] and •[5] apply to the entire curve.
7. In each of the following circumstances :
   - Candidates who transform the original graph
   - Candidates who sketch a parabola in (a)

   mark the candidate's attempt as normal and unless a mark of 0 has been scored, deduct the last mark awarded. Indicate this with (see Regular occurring response G).
8. A reflection in any line parallel to the y-axis does not gain •[4] or •[6] .
9. A translation other than $\begin{bmatrix} 0 \\ 2 \end{bmatrix}$ does not gain •[5] or •[6] .

Graph for (a)



Graph for (b)



Page 19

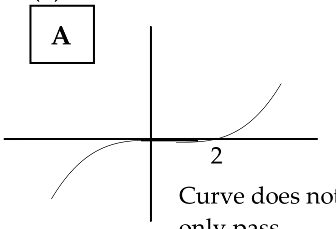
•[4] ic reflection in $x-$axis

4 continued

Regularly occurring responses

In (a)

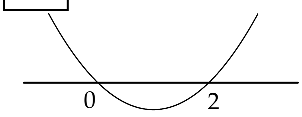
A



Curve does not only pass through 0 and 2 on x-axis.

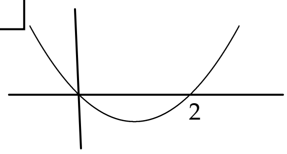
✗ $\bullet^1$ ✗ $\bullet^2$ ✗ $\bullet^3$

B



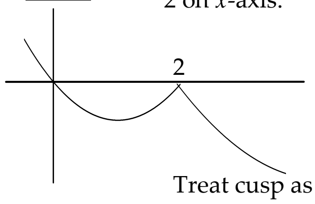
✗ $\bullet^1$ ✗ $\bullet^2$ ✗ $\bullet^3$

C



✓ $\bullet^1$ ✗ $\bullet^2$ ✗ $\bullet^3$

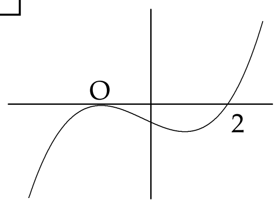
D



Treat cusp as turning point.

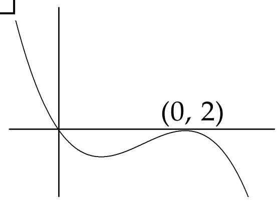
✓ $\bullet^1$ ✓ $\bullet^2$ ✓ $\bullet^3$

E



✗ $\bullet^1$ ✗ $\bullet^2$ ✗ $\bullet^3$

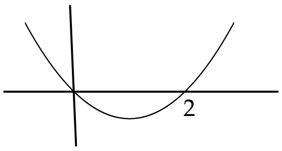
F



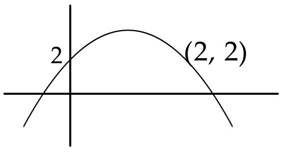
✓ $\bullet^1$ ✓ $\bullet^2$ ✓ $\bullet^3$

In (a) and (b)

G



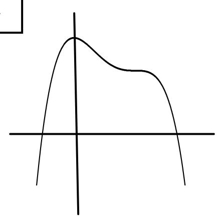
✓ $\bullet^1$ ✗ $\bullet^2$ ✗ $\bullet^3$



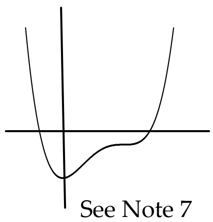
See Note 7

✓/ $\bullet^4$ ✓/ $\bullet^5$ ✓// $\bullet^6$

H



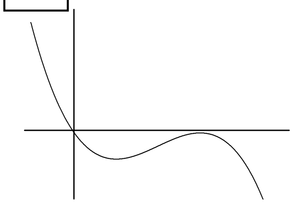
✗ $\bullet^1$ ✗ $\bullet^2$ ✗ $\bullet^3$



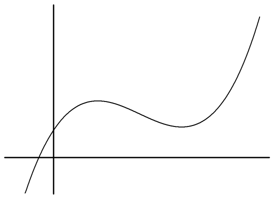
See Note 7

✓// $\bullet^4$ ✗ $\bullet^5$ ✗ $\bullet^6$

I

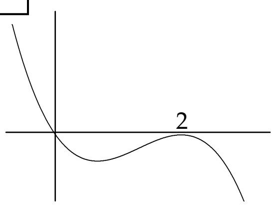


✗ $\bullet^1$ ✗ $\bullet^2$ ✓ $\bullet^3$

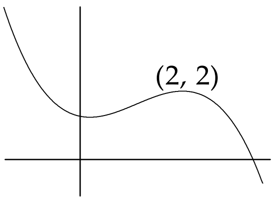


✓ $\bullet^4$ ✗ $\bullet^5$ ✗ $\bullet^6$

J



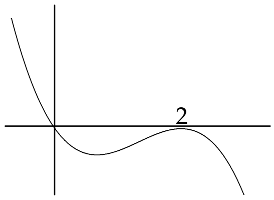
✓ $\bullet^1$ ✓ $\bullet^2$ ✓ $\bullet^3$



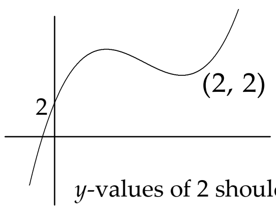
See Note 6

✗ $\bullet^4$ ✗ $\bullet^5$ ✗ $\bullet^6$

K



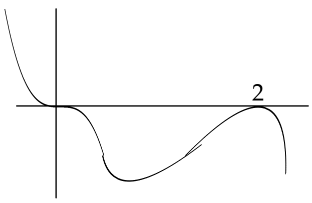
✓ $\bullet^1$ ✓ $\bullet^2$ ✓ $\bullet^3$



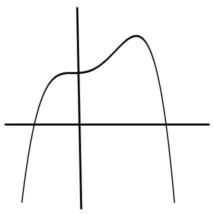
y-values of 2 should be roughly in line – too far apart here

✓ $\bullet^4$ ✓ $\bullet^5$ ✗ $\bullet^6$

L



✓ $\bullet^1$ ✓ $\bullet^2$ ✗ $\bullet^3$



✗ $\bullet^4$ ✗ $\bullet^5$ ✗ $\bullet^6$

5 A is the point $(3, -3, 0)$, B is $(2, -3, 1)$ and C is $(4, k, 0)$.

(a) (i) Express $\overrightarrow{BA}$ and $\overrightarrow{BC}$ in component form.

(ii) Show that $\cos A\hat{B}C = \frac{3}{\sqrt{2(k^2+6k+14)}}$. **7**

Generic Scheme			Illustrative Scheme
5(a)			
•1	ic	interpret vector	•1 $\begin{pmatrix}1\\0\\-1\end{pmatrix}$ Treat $\begin{pmatrix}1\\0\\-1\end{pmatrix}$ written as $(1, 0, -1)$ as bad form.
•2	pd	process vector	•2 $\begin{pmatrix}2\\k+3\\-1\end{pmatrix}$
•3	ss	use scalar product	•3 $\cos A\hat{B}C = \frac{\overrightarrow{BA}.\overrightarrow{BC}}{\lvert\overrightarrow{BA}\rvert\lvert\overrightarrow{BC}\rvert}$ see Note 1
•4	pd	find scalar product	•4 3
•5	pd	find $\lvert\overrightarrow{BA}\rvert$	•5 $\sqrt{2}$
•6	ic	find expression for $\lvert\overrightarrow{BC}\rvert$	•6 $\sqrt{2^2+(k+3)^2+(-1)^2}$ or equivalent
•7	ic	complete to result	•7 $\frac{3}{\sqrt{2}\sqrt{k^2+6k+14}}$ **and** $\frac{3}{\sqrt{2(k^2+6k+14)}}$ **or** $\lvert\overrightarrow{BA}\rvert\lvert\overrightarrow{BC}\rvert = \sqrt{2}\times\sqrt{k^2+6k+14}$ **and** $\frac{3}{\sqrt{2(k^2+6k+14)}}$

7

Notes

1. If the evidence for •3 does not appear explicitly, then •3 is only awarded if working for •7 is attempted.
2. •7 is dependent on gaining •4, •5 and •6.

Regularly occurring responses

Response 1 : Calculating wrong angle

Candidate A

$\cos AOC = \frac{\overrightarrow{OA}.\overrightarrow{OC}}{\lvert\overrightarrow{OA}\rvert\lvert\overrightarrow{OC}\rvert}$ ✗ •3

$\overrightarrow{OA}.\overrightarrow{OC} = 3\times4+(-3)\times k+0\times0 = 12-3k$ ✗ •4

$\lvert\overrightarrow{OA}\rvert = \sqrt{18}$ ✗ •5

$\lvert\overrightarrow{OC}\rvert = \sqrt{16+k^2}$ ✗ •6

$\cos ABC = \frac{12-3k}{\sqrt{18}\sqrt{16+k^2}}$ ✗ •7

Candidate B

$\cos AOB = \frac{\overrightarrow{OA}.\overrightarrow{OB}}{\lvert\overrightarrow{OA}\rvert\lvert\overrightarrow{OB}\rvert}$ ✗ •3

$\overrightarrow{OA}.\overrightarrow{OB} = 3\times2+(-3)\times(-3)+0\times1 = 15$ ✗ •4

$\lvert\overrightarrow{OA}\rvert = \sqrt{18}$ ✗ •5

$\lvert\overrightarrow{OB}\rvert = \sqrt{14}$ ✗ •6

$\cos ABC = \frac{15}{\sqrt{18}\sqrt{14}}$ ✗ •7

5 (b) If angle $ABC = 30^\circ$, find the possible values of k.	5

Generic Scheme | **Illustrative Scheme**

5(b)

Generic Scheme	Illustrative Scheme
Method 1 : Squaring first	Method 1 : Squaring first
•8 ic link with (a)	•8 $\frac{3}{\sqrt{2(k^2+6k+14)}} = \cos 30^\circ$
•9 ss square both sides	•9 $\left(\frac{3}{\sqrt{2(k^2+6k+14)}}\right)^2 = \left(\frac{\sqrt{3}}{2}\right)^2$
•10 pd rearrange into 'non-fractional' format	•10 $k^2+6k+14=6$ or equivalent
•11 pd write in standard form	•11 $k^2+6k+8=0$ or equivalent
•12 pd solve for k	•12 $k=-2$ or -4
Method 2 : Dealing with fractions first	Method 2 : Dealing with fractions first
•8 ic link with (a)	•8 $\frac{3}{\sqrt{2(k^2+6k+14)}} = \cos 30^\circ$
•9 pd rearrange into 'non-fractional' format	•9 $\sqrt{3}\sqrt{2(k^2+6k+14)}=6$
•10 ss square both sides	•10 $6(k^2+6k+14)=36$
•11 pd write in standard form	•11 $k^2+6k+8=0$ or equivalent
•12 pd solve for k	•12 $k=-2$ or -4

(Method 1 annotation: $= 0$ must appear at this stage.)

(Method 2 annotation: $= 0$ must appear at this stage.)

5

Notes

3. The evidence for •9 may appear in the working for •10 in both methods.

4. •9 is the only mark available to candidates who replace cos 30° by 30 in method 1 and •10 in method 2.

5. All 5 marks are available to candidates who use 0·87 for cos 30° but 0·9 can gain a maximum of 4 marks.

Regularly occurring responses

Response 2 : Working with cos 30° throughout the question

Candidate C (Method 1)

$\cos 30^\circ = \frac{3}{\sqrt{2(k^2+6k+14)}}$ ✓ •8

$(\cos 30^\circ)^2 = \left(\frac{3}{\sqrt{2(k^2+6k+14)}}\right)^2$ ✓ •9

$(\cos 30^\circ)^2 = \frac{9}{2(k^2+6k+14)}$

$2(\cos 30^\circ)^2(k^2+6k+14)=9$ ✓ •10

If cos 30° is subsequently evaluated then •11 and •12 may still be available.

Response 3 : Using the wrong value for cos 30°

Candidate D (Method 2)

$\frac{3}{\sqrt{2(k^2+6k+14)}} = \frac{1}{2}$ ✗ •8

$\sqrt{2(k^2+6k+14)}=6$ ✗ •9

$2(k^2+6k+14)=36$ ✗ •10

$k^2+6k+14=18$

$k^2+6k-4=0$ ✗ •11

$k = \frac{-6\pm\sqrt{6^2-4\times1\times(-4)}}{2\times1}$

$=0\cdot61, -6\cdot61$ ✗ •12

6 For $0 < x < \frac{\pi}{2}$, sequences can be generated using the recurrence relation

$$u_{n+1} = (\sin x)u_n + \cos 2x, \text{ with } u_0 = 1.$$

(a) Why do these sequences have a limit? **2**

Generic Scheme | **Illustrative Scheme**

6 (a)

•1	ic	condition on u_n coefficient	•1 $-1 < \sin x < 1$
•2	ic	connect coefficient with given interval	•2 in interval, $0 < \sin x < 1$

2

Notes

1. For •1 **do not** accept:
   - $\sin x$ lies between -1 and 1
   - $-1 < x < 1$
   - $-1 < \sin < 1$

   However, accept '$\sin x$ greater than -1 and less than 1' for •1.
2. Do not accept $-1 < a < 1$ for •1 unless a is clearly identified as $\sin x$, which may not appear until (b).
3. $0 < \sin x < 1$ and nothing else, does not gain •1 but gains •2.
4. $0 \le \sin x \le 1$ and nothing else, does not gain •1 or •2.

Regularly occurring responses

Response 1 : Attempts at giving a reason for limit

Candidate A

This sequence has a limit because $-1 < a < 1$, — •1 ✓

$-1 < \sin x < 1$ within the domain. — •2 ✗

Candidate B

Since $\sin x$ in this domain will always — •1 ✗

be greater than 0 and less than 1. — •2 ✓

Candidate C

$\sin\frac{\pi}{2} = 1$ and $\sin 0 = 0$ so the multiplier — •1 ✗

of u_n is between 0 and 1, so it has a limit. — •2 ✗

Candidate D

$-1 \le \sin x \le 1$,

for $0 < x < \frac{\pi}{2}$, $0 < \sin x < 1$ — •1 ✗

so limit exists — •2 ✓

Response 2 : Minimum response for both marks

Candidate E

for $0 < x < \frac{\pi}{2}$, $0 < \sin x < 1$ •2 ✓

so $-1 < \sin x < 1$ •1 ✓

so limit

Candidate F

if limit, $-1 < \sin x < 1$ •1 ✓

for $0 < x < \frac{\pi}{2}$, $0 < \sin x < 1$ •2 ✓

6	(b)	The limit of one particular sequence generated by this recurrence relation is $\frac{1}{2}\sin x$. Find the value(s) of x. **7**

Generic Scheme			**Illustrative Scheme**
6 (b)			
$\bullet^3$	ss	appropriate limit method	$\bullet^3$ limit $= \dfrac{\cos 2x}{1-\sin x}$ **or** $l = \sin x \times l + \cos 2x$
$\bullet^4$	ic	substitute for limit	$\bullet^4$ $\frac{1}{2}\sin x = \dfrac{\cos 2x}{1-\sin x}$ **or** $\frac{1}{2}\sin x = \sin x \times \frac{1}{2}\sin x + \cos 2x$
			($\bullet^3$ **may be stated, or implied by** $\bullet^4$ in both methods)
$\bullet^5$	ss	use appropriate double angle formula	$\bullet^5$ $\ldots 1-2\sin^2 x \ldots$
$\bullet^6$	pd	express in standard form	$\bullet^6$ e.g. $3\sin^2 x + \sin x - 2$ } $=0$ must appear at $\bullet^6$ or $\bullet^7$ to gain $\bullet^6$.
$\bullet^7$	pd	start to solve quadratic equation	$\bullet^7$ e.g. $(3\sin x - 2)(\sin x + 1)$
$\bullet^8$	pd	reduce to equations in $\sin x$ only	$\bullet^8$ $\sin x = \frac{2}{3}$ or $\sin x = -1$
$\bullet^9$	ic	select valid solution	$\bullet^9$ $x = 0\cdot730$ or outwith interval **7**

Notes

5. $\bullet^7$, $\bullet^8$ and $\bullet^9$ are only available if a quadratic equation is obtained at $\bullet^6$ stage.
6. Candidates may express the quadratic equation at the $\bullet^6$ stage in the form $3s^2 + s - 2 = 0$. For candidates who do not solve a trigonometric quadratic equation at $\bullet^7$ $\sin x$ must appear explicitly to gain $\bullet^8$.
7. $\bullet^7$, $\bullet^8$ and $\bullet^9$ are not available to candidates who 'solve' a quadratic equation in the form $ax^2 + bx = c$, $c \neq 0$.
8. For $\bullet^9$ there must be one valid solution, and one solution outwith interval which is rejected.
9. $\bullet^9$ is not available to candidates who leave their answer in degree measure.
10. Cross marking is available for $\bullet^8$ and $\bullet^9$.

Regularly occurring responses

Response 3 : Evidence for identification of a appearing in (b)

Candidate G

(a) $-1 < a < 1$ ^ — $\bullet^1$ ✓, $\bullet^2$ ✗

(b) $L = \dfrac{b}{1-a} = \dfrac{\cos 2x}{1-\sin x}$ ✓ $\bullet^3$ ✓ $\bullet^1$ — $\bullet^3$ ✓

Response 4 : Error in algebra and subsequent quadratic equation solution

Candidate H

$L = \dfrac{b}{1-a} = \frac{1}{2}\sin x$

$\dfrac{\cos 2x}{1-\sin x} = \frac{1}{2}\sin x$ ✓ $\bullet^3$ ✓ $\bullet^4$

$\cos 2x = -\frac{1}{2}\sin^2 x$ ✗ $\bullet^6$

$\frac{1}{2}\sin^2 x + \cos 2x = 0$

$\frac{1}{2}\sin^2 x + (1 - 2\sin^2 x) = 0$ ✓̸ $\bullet^5$

$-\frac{3}{2}\sin^2 x + 1 = 0$

$\sin^2 x = \frac{2}{3}$ ✓̸ $\bullet^7$

$\sin x = \sqrt{\frac{2}{3}}$ and $\sin x = -\sqrt{\frac{2}{3}}$ ✓̸ $\bullet^8$

$x = 0\cdot955$, ~~$2\cdot186$~~ $x = 4$~~$\cdot097$~~, ~~$5\cdot328$~~ ✓̸ $\bullet^9$

Candidate I

$\dfrac{\cos 2x}{1-\sin x} = \frac{1}{2}\sin x$ ✓ $\bullet^3$ ✓ $\bullet^4$

$\frac{1}{2}\sin x(1-\sin x) = 1 - \sin^2 x$ ✗ $\bullet^5$

$\sin^2 x + \sin x - 2 = 0$ ✓̸ $\bullet^6$

$(\sin x - 1)(\sin x + 2) = 0$ ✓̸ $\bullet^7$

$\sin x = 1$ and $\sin x = -2$ ✓̸ $\bullet^8$

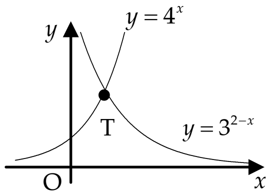
$x = \frac{\pi}{2}$ not possible ✓̸ $\bullet^9$

See Note 8

7 The diagram shows the curves with equations $y = 4^x$ and $y = 3^{2-x}$.

The graphs intersect at the point T.

(a) Show that the x-coordinate of T can be written in the form $\frac{\log_a p}{\log_a q}$, for all $a > 1$. **6**



Generic Scheme	Illustrative Scheme

7(a)

Generic Scheme			Illustrative Scheme (Method 1)	
•1	ss	equate expressions for y	•1 $4^x = 3^{2-x}$	
•2	ss	take logarithms of both sides	•2 $\log_a\left(4^x\right) = \log_a\left(3^{2-x}\right)$	stated, or implied by •3
•3	ic	use law of logs : $\log_a x^n = n\log_a x$	•3 $x\log_a 4 = (2-x)\log_a 3$	
•4	pd	gather like terms	•4 $x(\log_a 4 + \log_a 3) = 2\log_a 3$	
•5	ic	use law of logs : $\log_a p + \log_a q = \log_a pq$	•5 $x\log_a 12 = \log_a 9$	
•6	ic	complete to required form	•6 $\frac{\log_a 9}{\log_a 12}$	stated explicitly

Method 2

- •1 $4^x = 3^{2-x}$
- •2 $\log_3\left(4^x\right) = 2 - x$
- •3 $x\log_3 4 = 2 - x$
- •4 $x = \frac{2}{1+\log_3 4}$
- •5 $\frac{2\log_3 3}{\log_3 12}$
- •6 $\frac{\log_a 9}{\log_a 12}$ **stated explicitly**

Method 3

- •1 $4^x = 3^{2-x}$
- •2 $4^x = \frac{3^2}{3^x}$
- •3 $12^x = 9$
- •4 $\log_a 12^x = \log_a 9$
- •5 $x\log_a 12 = \log_a 9$
- •6 $\frac{\log_a 9}{\log_a 12}$ **stated explicitly**

6

In methods 1 and 2:

If the first line of working is that at the •2 stage, then •1 and •2 are awarded.

If the first line of working is that at the •3 stage, then only •2 and •3 are awarded.

Notes

1. In methods 1 and 2, if no base is indicated then •2 is not available , however •3, •4 and •5 are still available.

   In method 3, if no base is indicated then •4 is not available , however •5 is still available.
2. In all methods, if a numerical base is used then •6 is not available.
3. In method 1, the omission of brackets at the •3 stage is treated as bad form, see Response 1.
4. p and q must be numerical values.

Regularly occurring responses

Response 1 : Omission of brackets around $2-x$

Candidate A

$4^x = 3^{2-x}$ ✓ •1

$x\log_a 4 = 2 - x\log_a 3$ ✓ •2 ✓ •3

Candidate B

$4^x = 3^{2-x}$ ✓ •1

$x\log_a 4 = 2 - x\log_a 3$ ✓ •2 ✓ •3

$x(\log_a 4 + \log_a 3) = 2$ ✗ •4

$x\log_a 12 = 2$ ✗ •5

Response 2 : Using different bases

Candidate C

$4^x = 3^{2-x}$ ✓ •1

$\log_3 4^x = \log_4 3^{2-x}$ ✗ •2

$x\log_3 4 = (2-x)\log_4 3$ ✗ •3

Response 3 : Taking logs first

Candidate D

$y = 4^x$ and $y = 3^{2-x}$

$\log_a y = \log_a 4^x$ and $\log_a y = \log_a 3^{2-x}$ ✓ •2

$\log_a y = x\log_a 4$ and $\log_a y = (2-x)\log_a 3$ ✓ •3

$x\log_a 4 = (2-x)\log_a 3$ ✓ •1

$x = \frac{2}{\log_a 12}$

$= \frac{2\log_a a}{\log_a 12}$

$= \frac{\log_a a^2}{\log_a 12}$ ✗ •6

7	(b)	Calculate the y-coordinate of T.	2

Generic Scheme | **Illustrative Scheme**

7(b)

Generic Scheme		Illustrative Scheme	
$\bullet^7$ ic substitute in for x	$\bullet^7$ e.g. $y = 4^{\frac{\log_a 9}{\log_a 12}}$	**stated, or implied by $\bullet^8$**
$\bullet^8$ pd process y	$\bullet^8$ e.g. $y \approx 4^{0\cdot8842} \approx 3\cdot4$	2

Notes

5. Candidates must work to at least two significant figures in (b) e.g. $4^{0\cdot9} = 3\cdot5$ does not gain $\bullet^8$, but $\bullet^7$ is available.

6. $\bullet^8$ is only available if the power used comes from $\dfrac{\log_a p}{\log_a q}$ in (a).

Regularly occurring responses

Response 4 : Using p and q as integer values without working

Candidate E

$\left.\begin{array}{l} p=4 \\ q=3 \end{array}\right\} y = 4^{1\cdot26} = 5\cdot74 \text{ or } 5\cdot75$ ✗ $\bullet^7$ ✗ $\bullet^8$

Candidate F

$\left.\begin{array}{l} p=3 \\ q=4 \end{array}\right\} y = 4^{0\cdot79} = 2\cdot99 \text{ or } 3$ ✗ $\bullet^7$ ✗ $\bullet^8$

Response 5 : Using integer values calculated in (a)

Candidate G

$\left.\begin{array}{l} p=10 \\ q=4 \end{array}\right\} y = 4^{2\cdot5} = 32$ ✗ $\bullet^7$ ✗ $\bullet^8$

[END OF MARKING INSTRUCTIONS]