

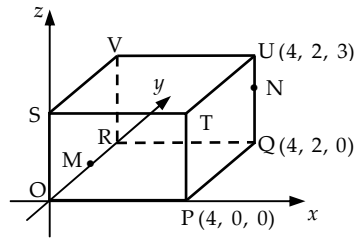
1 The diagram shows a cuboid OPQR,STUV relative to the coordinate axes.

P is the point (4, 0, 0),

Q is (4, 2, 0) and U is (4, 2, 3).

M is the midpoint of OR.

N is the point on UQ such that $UN = \frac{1}{3}UQ$.



(a) State the coordinates of M and N. 2

(b) Express the vectors \overline{VM} and \overline{VN} in component form. 2

Treat as bad form, coordinates written as components and vice versa, throughout this question.

Generic Scheme

Illustrative Scheme

1 (a)

- ¹ ic interpret midpoint for M
- ² ic interpret ratio for N

- ¹ (0, 1, 0)
- ² (4, 2, 2)

1 (b)

- ³ ic interpret diagram
- ⁴ pd process vectors

- ³ $\overline{VM} = \begin{pmatrix} 0 \\ -1 \\ -3 \end{pmatrix}$
- ⁴ $\overline{VN} = \begin{pmatrix} 4 \\ 0 \\ -1 \end{pmatrix}$

Using evidence from (a) or may have been taken directly from diagram.

Notes

1. V is the point (0, 2, 3), which may or may not appear in the working to (b).

Regularly occurring responses

Response 1

(a) $M(2, 0, 0) \times$ •¹ $N(4, 2, -1) \times$ •²

0 marks out of 2

(b) $\overline{VM} = \begin{pmatrix} 2 \\ -2 \\ -3 \end{pmatrix} \times$ •³

Consistent with (a)

$\overline{VN} = \begin{pmatrix} 4 \\ 0 \\ -1 \end{pmatrix} \checkmark$ •⁴

From diagram

2 marks out of 2

Response 2

(b) $V(0, 3, 2)$

Incorrect V stated

$\overline{VM} = \begin{pmatrix} 0 \\ -2 \\ -2 \end{pmatrix} \times$ •³

$\overline{VN} = \begin{pmatrix} 4 \\ -1 \\ 0 \end{pmatrix} \times$ •⁴

1 mark out of 2

Response 3

(a) $M(0, 2, 0) \times$ •¹ $N(4, 2, 2) \checkmark$ •²

1 mark out of 2

(b) $\overline{VM} = \begin{pmatrix} -4 \\ 0 \\ -3 \end{pmatrix} \times$ •³

$\overline{VN} = \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix} \times$ •⁴

V(4, 2, 3) used in both but not stated

0 marks out of 2

1 The diagram shows a cuboid OPQR,STUV relative to the coordinate axes.

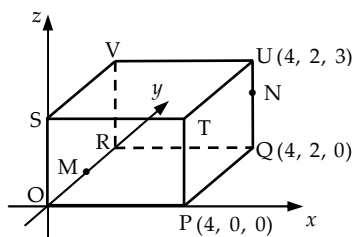
P is the point (4, 0, 0),

Q is (4, 2, 0) and U is (4, 2, 3).

M is the midpoint of OR.

N is the point on UQ such that $UN = \frac{1}{3}UQ$.

(c) Calculate the size of angle MVN.



5

Treat as bad form, coordinates written as components and vice versa, throughout this question.

Generic Scheme

Illustrative Scheme

1 (c)

Method 1 : Vector Approach

- ⁵ ss know to use scalar product
- ⁶ pd find scalar product
- ⁷ pd find magnitude of a vector
- ⁸ pd find magnitude of a vector
- ⁹ pd evaluate angle

Method 2 : Cosine Rule Approach

- ⁵ ss know to use cosine rule
- ⁶ pd find magnitude of a side
- ⁷ pd find magnitude of a side
- ⁸ pd find magnitude of a side
- ⁹ pd evaluate angle

Method 1 : Vector Approach

- ⁵ $\cos \hat{M}\hat{V}\hat{N} = \frac{\overline{VM} \cdot \overline{VN}}{|\overline{VM}| |\overline{VN}|}$ *stated, or implied by* •⁹
- ⁶ $\overline{VM} \cdot \overline{VN} = 3$
- ⁷ $|\overline{VM}| = \sqrt{10}$
- ⁸ $|\overline{VN}| = \sqrt{17}$
- ⁹ 76.7° or 1.339 rads or 85.2 grads

Method 2 : Cosine Rule Approach

- ⁵ $\cos \hat{M}\hat{V}\hat{N} = \frac{VM^2 + VN^2 - MN^2}{2 \times VM \times VN}$ *stated, or implied by* •⁹
- ⁶ $VM = \sqrt{10}$
- ⁷ $VN = \sqrt{17}$
- ⁸ $MN = \sqrt{21}$
- ⁹ 76.7° or 1.339 rads or 85.2 grads

Notes

2. •⁵ is not available to candidates who choose to evaluate an incorrect angle.
3. For candidates who do not attempt •⁹, then •⁵ is only available if the formula quoted relates to the labelling in the question.
4. •⁹ should be awarded for any answer that rounds to 77° or 1.3 rads or 85 grads (i.e. correct to two significant figures.)

Regularly occurring responses

Response 1

$\cos \hat{M}\hat{O}\hat{N} = \frac{\overline{OM} \cdot \overline{ON}}{|\overline{OM}| |\overline{ON}|}$ ✗ •⁵ Wrong angle

$\overline{OM} \cdot \overline{ON} = 2$ ✓ •⁶
 $|\overline{OM}| = 1$ ✗ •⁷ Eased because only one non-zero component.
 $|\overline{ON}| = \sqrt{24}$ ✗ •⁸

65.9° or 1.150 rads or 73.2 grads ✗ •⁹ 3 marks out of 5

Response 2

$\cos \hat{M}\hat{V}\hat{N} = \frac{\overline{VM} \cdot \overline{VN}}{|\overline{VM}| |\overline{VN}|}$ ✓ •⁵ Going directly to 90° from •⁶ would lose •⁷ and •⁸.

$\overline{VM} \cdot \overline{VN} = 0$ ✓ •⁶
 $|\overline{VM}| = \sqrt{17}$ ✗ •⁷
 $|\overline{VN}| = 2$ ✗ •⁸

90° or equivalent ✗ •⁹ 4 marks out of 5

2 (a) $12 \cos x^\circ - 5 \sin x^\circ$ can be expressed in the form $k \cos(x+a)^\circ$, where $k > 0$ and $0 \leq a < 360$.
Calculate the values of k and a .

4

Generic Scheme

Illustrative Scheme

(a)

- ¹ ss use addition formula
- ² ic compare coefficients
- ³ pd process k
- ⁴ pd process a

- ¹ $k \cos x^\circ \cos a^\circ - k \sin x^\circ \sin a^\circ$ or $k(\cos x^\circ \cos a^\circ - \sin x^\circ \sin a^\circ)$
stated explicitly
- ² $k \cos a^\circ = 12$ and $k \sin a^\circ = 5$ or $-k \sin a^\circ = -5$
stated explicitly
- ³ 13 *no justification required, but do not accept $\sqrt{169}$*
- ⁴ 22.6 *accept any answer which rounds to 23*

Notes

- Do not penalise the omission of the degree symbol.
- Treat $k \cos x^\circ \cos a^\circ - \sin x^\circ \sin a^\circ$ as bad form only if the equations at the •² stage both contain k .
- $13 \cos x^\circ \cos a^\circ - 13 \sin x^\circ \sin a^\circ$ or $13(\cos x^\circ \cos a^\circ - \sin x^\circ \sin a^\circ)$ is acceptable for •¹ and •³.
- ² is not available for $k \cos x^\circ = 12$ and $k \sin x^\circ = 5$ or $-k \sin x^\circ = -5$, however, •⁴ is still available.
- ⁴ is lost to candidates who give a in radians only.
- ⁴ may be gained only as a consequence of using evidence at •² stage.
- Candidates may use any form of the wave equation for •¹, •² and •³, however •⁴ is only available if the value of a is interpreted for the form $k \cos(x+a)^\circ$.

Regularly occurring responses

Response 1A

$k(\cos x^\circ \cos a^\circ - \sin x^\circ \sin a^\circ) \checkmark$ •¹
 $\sin a = 5 \quad \times$ •²
 $\cos a = 12$
 $\tan a^\circ = \frac{5}{12}$
 $a = 22.6 \quad \times$ •⁴
 $13 \cos(x + 22.6)$
 \checkmark •³

2 marks out of 4

Response 1B

$k \cos x \cos a - k \sin x \sin a \checkmark$ •¹
 $k = 13 \checkmark$ •³ \wedge •²
 $\tan a^\circ = \frac{5}{12}$
 $a = 22.6 \quad \times$ •⁴

2 marks out of 4

Response 2

$k \cos(x-a)$
 $= k \cos x \cos a + k \sin x \sin a \quad \times$ •¹
 $= 13 \cos x \cos a + 13 \sin x \sin a \quad \checkmark$ •³
 $13 \cos a = 12 \quad 13 \sin a = -5 \quad \times$ •²
 then $a = 22.6 \quad \times$ •⁴ See note 6
 or $a = 337.4 \quad \times$ •⁴ See note 7

3 marks out of 4

Response 3A

$k \cos x^\circ \cos a^\circ - \sin x^\circ \sin a^\circ$
 $k \sin a = 5 \quad \checkmark$ •¹ \checkmark •²
 $k \cos a = 12$
 $k = 13 \quad \checkmark$ •³ $\tan a^\circ = \frac{12}{5} \quad \times$ •⁴
 $a = 67.4$

3 marks out of 4

Response 3B

$k \cos x^\circ \cos a^\circ - \sin x^\circ \sin a^\circ$
 $k \sin a = 12 \quad \checkmark$ •¹ \times •²
 $k \cos a = 5$
 $k = 13 \quad \checkmark$ •³ $\tan a^\circ = \frac{12}{5}$
 $a = 67.4 \quad \times$ •⁴

3 marks out of 4

Response 4

$k \cos x^\circ \cos a^\circ - k \sin x^\circ \sin a^\circ \checkmark$ •¹
 $k \cos a = 12 \quad \times$ •²
 $k \sin a = -5$
 $k = 13 \quad \checkmark$ •³ $\tan a^\circ = -\frac{5}{12}$
 $a = 337.4 \quad \times$ •⁴ See note 6

3 marks out of 4

- 2 (b) (i) Hence state the maximum and minimum values of $12 \cos x^\circ - 5 \sin x^\circ$.
 (ii) Determine the values of x , in the interval $0 \leq x < 360$, at which these maximum and minimum values occur.

3

Generic Scheme

Illustrative Scheme

(b)

- ⁵ ss state maximum and minimum
- ⁶ ic find x corresponding to max. value
- ⁷ pd find x corresponding to min. value

- ⁵ 13, -13
- ⁶ maximum at $337 \cdot 4$ and no others
- ⁷ minimum at $157 \cdot 4$ and no others
- or
- ⁶ $337 \cdot 4$ and $157 \cdot 4$ and no others
- ⁷ maximum at $337 \cdot 4$ and minimum at $157 \cdot 4$

Notes

8. •⁵ is available for $\sqrt{169}$ and $-\sqrt{169}$ only if $\sqrt{169}$ has been penalised at •³.
9. Accept answers which round to 337 and 157 for •⁶ and •⁷.
10. Candidates who continue to work in radian measure should not be penalised further.
11. Extra solutions, correct or incorrect, should be penalised at •⁶ or •⁷ but not both.
12. •⁶ and •⁷ are not available to candidates who work with $13 \cos(x + 22 \cdot 6)^\circ = 0$ or $13 \cos(x + 22 \cdot 6)^\circ = 1$.
13. Candidates who use $13 \cos(x - 22 \cdot 6)^\circ$ from a correct (a) lose •⁶ but •⁷ is still available.

Regularly occurring responses

Response 1

From (a) $a = 67 \cdot 4$
 max/ min = ± 13 ✓ •⁵
 max at $292 \cdot 6$ ✗ •⁶
 min at $112 \cdot 6$ ✗ •⁷

3 marks out of 3

Response 2

From (a) $\sqrt{169} \cos(x + 22 \cdot 6)^\circ$
 max = $\sqrt{169}$ min = $-\sqrt{169}$ ✗ •⁵
 max at $22 \cdot 6$ ✗ •⁶
 min at $202 \cdot 6$ ✗ •⁷

2 marks out of 3

$\sqrt{169}$ already penalised at (a)

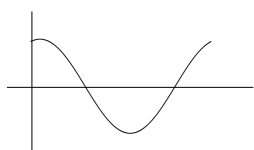
Response 3A

$13 \cos(x + 22 \cdot 6)^\circ$ max at $22 \cdot 6$ ✗ •⁶
 $\times 13$ $22 \cdot 6$ min at $202 \cdot 6$ ✗ •⁷

1 mark out of 3

Insufficient evidence for •⁵

Response 3B



$13 \cos(x + 22 \cdot 6)^\circ$ max at $22 \cdot 6$ ✗ •⁶
 min at $202 \cdot 6$ ✗ •⁷

1 mark out of 3

N.B. Candidates who use differentiation in (b) can gain •⁵ only, as a direct result of their response in (a). This question is in degrees and so calculus is not appropriate for •⁶ and •⁷.

3 (a) (i) Show that the line with equation $y = 3 - x$ is a tangent to the circle with equation $x^2 + y^2 + 14x + 4y - 19 = 0$.

(ii) Find the coordinates of the point of contact, P.

5

Generic Scheme

Illustrative Scheme

(a)

•¹ ss substitute

•² pd express in standard form

•³ ic start proof

•⁴ ic complete proof

•⁵ pd coordinates of P

$$\bullet^1 \quad x^2 + (3-x)^2 + 14x + 4(3-x) - 19 = 0$$

Method 1 : Factorising

$$\bullet^2 \quad 2x^2 + 4x + 2 \quad \left. \vphantom{2x^2 + 4x + 2} \right\} = 0 \quad \text{see note 1}$$

$$\bullet^3 \quad 2(x+1)(x+1)$$

•⁴ equal roots so line is a tangent

Method 2 : Discriminant

$$\bullet^2 \quad 2x^2 + 4x + 2 = 0 \quad \text{stated explicitly}$$

$$\bullet^3 \quad 4^2 - 4 \times 2 \times 2$$

$$\bullet^4 \quad b^2 - 4ac = 0 \text{ so line is a tangent}$$

$$\bullet^5 \quad x = -1, y = 4$$

Notes

For method 1 :

- ² is only available if “= 0” appears at either •² or •³ stage.
- Alternative wording for •⁴ could be e.g. ‘repeated roots’, ‘repeated factor’, ‘only one solution’, ‘only one point of contact’ **along with** ‘line is a tangent’.

For both methods :

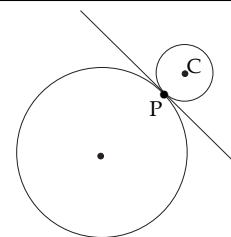
- Candidates must work with a quadratic equation at the •³ and •⁴ stages.
- Simply stating the tangency condition without supporting working cannot gain •⁴.
- For candidates who obtain two distinct roots, •⁴ is still available for ‘not equal roots so not a tangent’ or ‘ $b^2 - 4ac \neq 0$ so line is not a tangent’, but •⁵ is not available.

- 3 (b) Relative to a suitable set of coordinate axes, the diagram below shows the circle from (a) and a second smaller circle with centre C.

The line $y = 3 - x$ is a common tangent at the point P.

The radius of the larger circle is three times the radius of the smaller circle.

Find the equation of the smaller circle.



$$x^2 + y^2 + 14x + 4y - 19 = 0$$

6

Generic Scheme

Illustrative Scheme

(b)

Method 1 : via centre and radius

- ⁶ ic state centre of larger circle
- ⁷ ss find radius of larger circle
- ⁸ pd find radius of smaller circle
- ⁹ ss strategy for finding centre
- ¹⁰ ic interpret centre of smaller circle
- ¹¹ ic state equation

Method 2 : via ratios

- ⁶ ic state centre of larger circle
- ⁷ ss strategy for finding centre
- ⁸ ic state centre of smaller circle
- ⁹ ss strategy for finding radius
- ¹⁰ pd find radius of smaller circle
- ¹¹ ic state equation

Method 1 : via centre and radius

- ⁶ $(-7, -2)$ see note 11
- ⁷ $\sqrt{72}$ see note 6 stated, or implied by •⁸
- ⁸ $\sqrt{8}$ see note 7
- ⁹ e.g. "Stepping out"
- ¹⁰ $(1, 6)$
- ¹¹ $(x-1)^2 + (y-6)^2 = 8$ or $x^2 + y^2 - 2x - 12y + 29 = 0$

Method 2 : via ratios

- ⁶ $(-7, -2)$ see note 11
- ⁷ e.g. "Stepping out"
- ⁸ $(1, 6)$
- ⁹ $\sqrt{2^2 + 2^2}$
- ¹⁰ $\sqrt{8}$ see note 10
- ¹¹ $(x-1)^2 + (y-6)^2 = 8$ or $x^2 + y^2 - 2x - 12y + 29 = 0$

Notes

For method 1:

6. Acceptable alternatives for •⁷ are $6\sqrt{2}$ or decimal equivalent which rounds to 8.5 i.e. to two significant figures.
7. Acceptable alternatives for •⁸ are $\frac{\sqrt{72}}{3}$ or $2\sqrt{2}$ or decimal equivalent which rounds to 2.8.
8. $(1, 6)$ without working gains •¹⁰ but loses •⁹.

For method 2:

9. $(1, 6)$ without working gains •⁸ but loses •⁷.
10. Acceptable alternatives for •¹⁰ are $2\sqrt{2}$ or decimal equivalent which rounds to 2.8.

In both methods:

11. If $m = 1$ is used in a 'stepping out' method the centre of the larger circle need not be stated explicitly for •⁶.
12. For the smaller circle, candidates who 'guess' values for either the centre or radius cannot be awarded •¹¹.
13. At •¹¹ e.g. $\sqrt{8^2}$, $2 \cdot 8^2$ are unacceptable, but any decimal which rounds to 7.8 is acceptable.
14. •¹¹ is not available to candidates who divide the coordinates of the centre of the larger circle by 3.

Generic Scheme

Illustrative Scheme

4

- ¹ ss know to use double angle formula
- ² ic express as quadratic in $\cos x$
- ³ ss start to solve

- ⁴ pd reduce to equations in $\cos x$ only
- ⁵ pd complete solutions to include only one where $\cos x = k$ with $|k| > 1$

Method 1 : Using factorisation

- ¹ $2 \times (2 \cos^2 x - 1) \dots$
- ² $4 \cos^2 x - 5 \cos x - 6$
- ³ $(4 \cos x + 3)(\cos x - 2)$ } = 0 must appear at either of these lines to gain •².

Method 2 : Using quadratic formula

- ¹ $2 \times (2 \cos^2 x - 1) \dots$
- ² $4 \cos^2 x - 5 \cos x - 6 = 0$
- ³ $\cos x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4 \times 4 \times (-6)}}{2 \times 4}$

In both methods :

- ⁴ $\cos x = -\frac{3}{4}$ and $\cos x = 2$
- ⁵ $2 \cdot 419, 3 \cdot 864$ and no solution
- or**
- ⁴ $\cos x = 2$ and no solution
- ⁵ $\cos x = -\frac{3}{4}$ and $2 \cdot 419, 3 \cdot 864$

Notes

1. •¹ is not available for simply stating that $\cos 2A = 2 \cos^2 A - 1$ with no further working.
2. Substituting $\cos 2A = 2 \cos^2 A - 1$ or $\cos 2a = 2 \cos^2 a - 1$ etc. should be treated as bad form throughout.
3. In the event of $\cos^2 x - \sin^2 x$ or $1 - 2 \sin^2 x$ being substituted for $\cos 2x$, •¹ cannot be given until the equation reduces to a quadratic in $\cos x$.
4. Candidates may express the quadratic equation obtained at the •² stage in the form $4c^2 - 5c + 6 = 0$, $4x^2 - 5x + 6 = 0$ etc. For candidates who do not solve a trig. equation at •⁵, $\cos x$ must appear explicitly to gain •⁴.
5. •⁴ and •⁵ are only available as a consequence of solving a quadratic equation subsequent to a substitution.
6. Any attempt to solve $a \cos^2 x + b \cos x = c$ loses •³, •⁴ and •⁵.
7. Accept answers given as decimals which round to $2 \cdot 4$ and $3 \cdot 9$.
8. There must be an indication after $\cos x = 2$ that there are no solutions to this equation.
Acceptable evidence : e.g. " ~~$\cos x = 2$~~ ", "NA", "out of range", "invalid" and " $\cos x = 2$ no", " $\cos x = 2 \times$ "
Unacceptable evidence : e.g. " $\cos x = 2$ ", " $\cos x = 2$???", "Maths Error".
9. •⁵ is not available to candidates who work throughout in degrees and do not convert their answer into radian measure.
10. Do not accept e.g. $221 \cdot 4$, $138 \cdot 6$, $\frac{221 \cdot 4\pi}{180}$, $\frac{221\pi}{180}$, $1 \cdot 23\pi$.
11. Ignore correct solution outside the interval $0 \leq x < 2\pi$.

4

Regularly occurring responses

Response 1

$$2 \times 2 \cos^2 x - 1 \dots \checkmark \bullet^1$$

$$4 \cos^2 x - 5 \cos x - 5 = 0 \quad \times \bullet^2$$

$$\cos x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4 \times 4 \times (-5)}}{2 \times 4} \quad \times \bullet^3$$

$$\cos x = \frac{5 - \sqrt{105}}{8} \quad \text{and} \quad \cos x = \frac{5 + \sqrt{105}}{8} \quad \times \bullet^4$$

$$2 \cdot 286, 3 \cdot 997 \quad \text{and} \quad \text{no solution} \quad \times \bullet^5$$

\bullet^5 is only available to candidates where one, but not both, of their equations has no solution for $\cos x$.

4 marks out of 5

Response 2

$$4 \cos^2 x - 1 \dots \checkmark \bullet^1$$

$$4 \cos^2 x - 5 \cos x - 5 = 0 \quad \times \bullet^2$$

$$(2 \cos x + 1)(2 \cos x - 5) = 0 \quad \times \bullet^3$$

$$\cos x = -\frac{1}{2} \quad \text{and} \quad \cancel{\cos x = \frac{5}{2}} \quad \times \bullet^4$$

$$x = \frac{2\pi}{3}, \frac{4\pi}{3} \quad \times \bullet^5$$

$4 \cos^2 x - 1$ with no further working cannot gain \bullet^1 ; however if a quadratic in $\cos x$ subsequently appears then \bullet^1 is awarded but \bullet^2 is not available.

3 marks out of 5

\bullet^1 is lost here as it is not clear whether the candidate has used $2 \cos^2 x - 1$ or $\cos^2 x - 1$ as their substitution.

Response 3A

$$2 \cos^2 x - 1 - 5 \cos x - 4 = 0 \quad \times \bullet^1$$

$$2 \cos^2 x - 5 \cos x - 5 = 0 \quad \times \bullet^2$$

$$\cos x = \frac{5 \pm \sqrt{25 + 40}}{4} \quad \times \bullet^3$$

$$\cos x = -0.766 \quad \text{and} \quad \cos x = 3.267 \quad \times \bullet^4$$

$$x = 2.44, 3.84 \quad \text{and} \quad \text{undefined} \quad \times \bullet^5$$

4 marks out of 5

Response 3B

$$\cos 2x = 2 \cos^2 x - 1$$

$$2 \cos^2 x - 1 - 5 \cos x - 4 = 0 \quad \checkmark \bullet^1$$

$$2 \cos^2 x - 5 \cos x - 5 = 0 \quad \times \bullet^2$$

$$\cos x = \frac{5 \pm \sqrt{25 + 40}}{4} \quad \times \bullet^3$$

$$\cos x = -0.766 \quad \text{and} \quad \cos x = 3.267 \quad \times \bullet^4$$

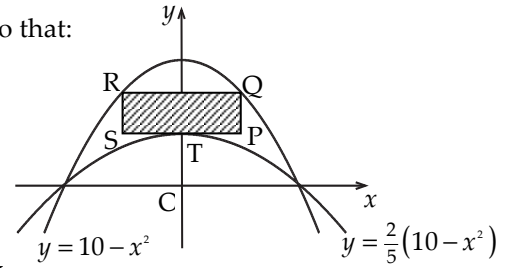
$$x = 2.44, 3.84 \quad \text{and} \quad \text{undefined} \quad \times \bullet^5$$

4 marks out of 5

5 The parabolas with equations $y = 10 - x^2$ and $y = \frac{2}{5}(10 - x^2)$ are shown in the diagram below.

A rectangle PQRS is placed between the two parabolas as shown, so that:

- Q and R lie on the upper parabola.
- RQ and SP are parallel to the x -axis.
- T, the turning point of the lower parabola, lies on SP.



- (a) (i) If $TP = x$ units, find an expression for the length of PQ.
 (ii) Hence show that the area, A , of rectangle PQRS is given by

$$A(x) = 12x - 2x^3 \quad 3$$

- (b) Find the maximum area of this rectangle. 6

Generic Scheme

Illustrative Scheme

5 (a)

- ¹ ss know to and find OT
- ² ic obtain an expression for PQ
- ³ ic complete area evaluation

- ¹ 4 or (0,4) *stated, or implied by* •²
- ² $10 - x^2 - 4$
- ³ $2x \times (6 - x^2) = 12x - 2x^3$

Notes

1. The evidence for •¹ and •² may appear on a sketch.
2. No marks are available to candidates who work backwards from the area formula.
3. •³ is only available if •² has been awarded.

5 (b)

- ⁴ ss know to and start to differentiate
- ⁵ pd complete differentiation
- ⁶ ic set derivative to zero
- ⁷ pd obtain x
- ⁸ ss justify nature of stationary point
- ⁹ ic interpret result and evaluate area

- ⁴ $A'(x) = 12 \dots$ *stated, or implied by* •⁵
- ⁵ $12 - 6x^2$
- ⁶ $12 - 6x^2 = 0$
- ⁷ $\sqrt{2}$ or decimal equivalent (ignore inclusion of $-\sqrt{2}$)
- ⁸

x	\dots	$\sqrt{2}$	\dots
$A'(x)$	$+$	0	$-$

(Note : accept $12 - 6x^2$ in lieu of $A'(x)$ in the nature table.)
- ⁹ Max **and** $8\sqrt{2}$ or decimal equivalent
 N.B. To conclude a maximum the evidence must come from •⁸.

Notes

4. At •⁷ accept any answer which rounds to 1.4.
5. Throughout this question treat the use of $f'(x)$ or $\frac{dy}{dx}$ as bad form.
6. At •⁸ the nature can be determined using the second derivative.
7. At •⁹ accept any answer which rounds to 11.3 or 11.4.

5

Regularly occurring responses

Response 1

$$A(x) = 12x - 2x^3$$

$$A'(x) = 24x^2 - 6x^3 \quad \times \bullet^4 \quad \times \bullet^5$$

$$24x^2 - 6x^3 = 0 \quad \times \bullet^6$$

$A'(x) = 0$ on its own would not be sufficient for \bullet^6 .

Response 2

At stationary points, $A'(x) = 0$

$$12 - 6x^2 \quad \checkmark \bullet^4 \quad \checkmark \bullet^5 \quad \checkmark \bullet^6$$

$$x = \sqrt{2} \quad \checkmark \bullet^7$$

Response 3

$$A(x) = 12x - 2x^3$$

$$= 12 - 6x^2 \quad \checkmark \bullet^4 \quad \checkmark \bullet^5$$

Bad form

Response 4A

	...	$\sqrt{2}$...
$A'(x)$	+	0	-

x missing

$\times \bullet^8$

Response 4B

x	1	$\sqrt{2}$	2
$A'(x)$	6	0	-12

$\checkmark \bullet^8$

Response 4C

x	...	$\sqrt{2}$...
$A'(x)$	/	-	\

or 'slope' etc.

signs or values are necessary

$\times \bullet^8$

Response 5

Maximum at $x = \sqrt{2}$

$$y = 12\sqrt{2} - 2\sqrt{2}^3 = 8\sqrt{2} \quad \checkmark \bullet^9$$

$$\text{Area} = 2\sqrt{2} \times 8\sqrt{2} = 32$$

Treat this as an error subsequent to a correct answer.

6 (a) A curve has equation $y = (2x - 9)^{\frac{1}{2}}$.

Show that the equation of the tangent to this curve at the point where $x = 9$ is $y = \frac{1}{3}x$.

5

(b) Diagram 1 shows part of the curve and the tangent.

The curve cuts the x -axis at the point A.

Find the coordinates of point A.

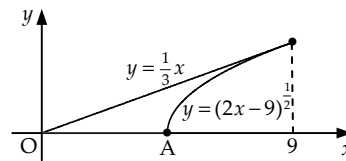


Diagram 1

1

Generic Scheme

Illustrative Scheme

6 (a)

- ¹ ss know to and start to differentiate
- ² pd complete chain rule derivative
- ³ pd gradient via differentiation
- ⁴ pd obtain y_{CURVE} at $x = 9$
- ⁵ ic state equation and complete

- ¹ $\frac{1}{2}(2x - 9)^{-\frac{1}{2}}$
- ² $\dots \times 2$
- ³ $\frac{1}{3}$
- ⁴ 3
- ⁵ $y - 3 = \frac{1}{3}(x - 9)$ and complete to $y = \frac{1}{3}x$

Notes

1. •³ is only available as a consequence of differentiating equation of the curve.
2. Candidates must arrive at the equation of the tangent via the point (9, 3) and not the origin.
3. For •³ accept $9^{-\frac{1}{2}}$.

Regularly occurring responses

Response 1

Candidates who equate derivatives:

$$\frac{1}{2}(2x - 9)^{-\frac{1}{2}} \times 2 = \frac{1}{3} \quad \checkmark \bullet^3$$

$\checkmark \bullet^1 \quad \checkmark \bullet^2$

leading to $x = 9$ and $y = 3$ from curve $\checkmark \bullet^4$

Also obtaining $y = 3$ from line and so line is a tangent $\checkmark \bullet^5$

5 marks out of 5

Response 2

Candidates who intersect curve and line:

$$(2x - 9)^{\frac{1}{2}} = \frac{1}{3}x \quad \checkmark \bullet^1$$

$$2x - 9 = \left(\frac{1}{3}x\right)^2 \quad \checkmark \bullet^2$$

$$\frac{1}{9}x^2 - 2x + 9 = 0 \quad \checkmark \bullet^3$$

Factorising or using discriminant $\checkmark \bullet^4$

Equal roots or $b^2 - 4ac = 0$ so line is a tangent $\checkmark \bullet^5$

5 marks out of 5

(b)

- ⁶ ic obtain coordinates of A

$\bullet^6 \left(\frac{9}{2}, 0\right)$

Notes

4. Accept $x = \frac{9}{2}, y = 0$ where $y = 0$ may appear from $(2x - 9)^{\frac{1}{2}} = 0$.
5. For $\left(\frac{9}{2}, 0\right)$ without working •⁶ is awarded, but from erroneous working •⁶ is lost (see response below).

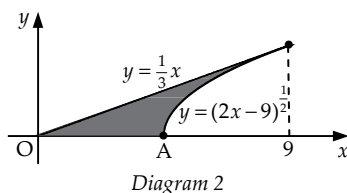
Regularly occurring responses

Response 1

$$\sqrt{2x - 9} = 0 \Rightarrow \sqrt{2x} - 3 = 0 \Rightarrow 2x - 9 = 0 \Rightarrow x = 4.5$$

Here •⁶ cannot be awarded due to the erroneous working.

6 (c) Calculate the shaded area shown in diagram 2.



7

Generic Scheme

Illustrative Scheme

6 (c)

Method 1 : Area of triangle – area under curve

- ⁷ ss strategy for finding shaded area
- ⁸ ss know to integrate $(2x - 9)^2$
- ⁹ pd start integration
- ¹⁰ pd complete integration
- ¹¹ ic limits x_A and 9
- ¹² pd substitute limits
- ¹³ pd evaluate area and complete strategy

Method 2 : Area between line and curve

- ⁷ ss strategy for finding shaded area
- ⁸ ss know to integrate $(2x - 9)^2$
- ⁹ pd start integration
- ¹⁰ pd complete integration
- ¹¹ ic limits x_A and 9
- ¹² pd 'upper – lower' and substitute limits
- ¹³ pd evaluate area and complete strategy

Method 1 : Area of triangle – area under curve

- ⁷ Shaded area = Area of large Δ – Area under curve
- ⁸ $\int (2x - 9)^2 dx$
- ⁹ $\frac{(2x - 9)^2}{\frac{3}{2}}$
- ¹⁰ $\dots \times \frac{1}{2}$
- ¹¹ $\frac{9}{2}$ and 9
- ¹² $\frac{1}{3}(18 - 9)^{\frac{3}{2}} - 0$
- ¹³ $\frac{27}{2} - 9 = \frac{9}{2}$ or $4\frac{1}{2}$ or 4.5

Method 2 : Area between line and curve

- ⁷ Area of small Δ + area between line and curve
- ⁸ $\int \dots (2x - 9)^2 dx$
- ⁹ $\dots \frac{(2x - 9)^2}{\frac{3}{2}}$
- ¹⁰ $\dots \times \frac{1}{2}$
- ¹¹ $\frac{9}{2}$ and 9
- ¹² $\left(\frac{1}{6} \times 9^2 - \frac{1}{3}(18 - 9)^{\frac{3}{2}} \right) - \left(\frac{1}{6} \times \left(\frac{9}{2} \right)^2 - \frac{1}{3}(9 - 9)^{\frac{3}{2}} \right)$
- ¹³ $\frac{27}{8} + \frac{9}{8} = \frac{9}{2}$ or $4\frac{1}{2}$ or 4.5

Notes

6. •⁷ may not be obvious until the final line of working and may be implied by final answer or a diagram.
7. At •¹¹ the value of x_A must lie between 0 and 9 exclusively, however, •¹² and •¹³ are only available if $4.5 \leq x_A < 9$.
8. Full marks are available to candidates who integrate with respect to y .

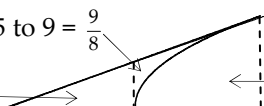
You may find the following helpful in marking this question:

Area between curve and line from 4.5 to 9 = $\frac{9}{8}$

Area of $\Delta_{\text{SMALLER}} = \frac{27}{8}$

Area of $\Delta_{\text{LARGER}} = 13.5$ or $\frac{27}{2}$

Area under curve from 4.5 to 9 = 9



Generic Scheme

Illustrative Scheme

(a)

- ¹ ss convert from log to exponential form
- ² ss know to and convert back to log form
- ³ pd process and complete

- ¹ $x = 4^P$
- ² $\log_{16} x = \log_{16} 4^P$
- ³ $\log_{16} x = P \times \log_{16} 4$ and complete

Notes

1. No marks are available to candidates who simply substitute in values and verify the result.

e.g. $\log_4 4 = 1$ and $\log_{16} 4 = \frac{1}{2}$ ✘
 $\log_4 x = P$ and $\log_{16} x = \frac{1}{2}P$

Regularly occurring responses

Response 1

$$\log_4 x = P$$

$$x = 4^P \quad \checkmark \bullet^1$$

$$\log_{16} x = \frac{1}{2}P \quad \text{✘} \bullet^2$$

$$x = 16^{\frac{1}{2}P}$$

$$= 4^P \quad \wedge \bullet^3$$

1 mark out of 3

Response 2

$$\log_4 x = P$$

$$x = 4^P \quad \checkmark \bullet^1$$

$$x^2 = 4^{2P}$$

$$= 16^P \quad \text{✘} \bullet^2$$

$$\log_{16} x^2 = P$$

$$2\log_{16} x = P$$

$$\log_{16} x = \frac{1}{2}P \quad \checkmark \bullet^3$$

2 marks out of 3

Response 3

$$x = 4^P \quad \checkmark \bullet^1$$

$$x = \left(16^{\frac{1}{2}}\right)^P \quad \text{or} \quad 16^{\frac{1}{2} \times P} \quad \checkmark \bullet^2$$

$$x = 16^{\frac{1}{2}P}$$

$$\log_{16} x = \frac{1}{2}P \quad \checkmark \bullet^3$$

Without this step \bullet^2 would be lost but \bullet^3 is still available as follow through.

3 marks out of 3

Response 4

$$x = 4^P \quad \checkmark \bullet^1 \quad \log_{16} x = kP$$

$$\log_4 x = P \quad 16^{\log_{16} x} = 16^{kP}$$

$$4^{\log_4 x} = 4^P = x \quad x = 16^{kP}$$

$$4^{\log_4 x} = x$$

$$4^P = 16^{kP}$$

$$4 = 16^k \quad \text{✘} \bullet^2$$

$$k = \frac{1}{2}$$

$$\therefore \log_{16} x = \frac{1}{2}P \text{ as } 16^{\frac{1}{2}} = 4 \quad \checkmark \bullet^3$$

2 marks out of 3

Response 5

Beware that some candidates give a circular argument.

This is only worth \bullet^1 .

$$\log_4 x = P \quad \text{then} \quad \log_{16} x = \frac{1}{2}P$$

$$x = 4^P \quad \checkmark \bullet^1 \quad x = 16^{\frac{1}{2}P}$$

$$\log_4 x = \log_4 4^P \quad \log_{16} x = \log_{16} 16^{\frac{1}{2}P}$$

$$\log_4 x = P \log_4 4 \quad \log_{16} x = \frac{1}{2}P \log_{16} 16$$

$$\log_4 x = P \quad \log_{16} x = \frac{1}{2}P \quad \text{✘}$$

1 mark out of 3

Response 6

$$\log_{16} x = \frac{\log_4 x}{\log_4 16} = \frac{\log_4 x}{2} = \frac{1}{2}P$$

$$\checkmark \bullet^1 \quad \checkmark \bullet^2 \quad \checkmark \bullet^3$$

Using change of base result.

3 mark out of 3

7 (b) Solve $\log_3 x + \log_9 x = 12$.

3

Generic Scheme

Illustrative Scheme

(b)

- ⁴ ss use appropriate strategy
- ⁵ pd start solving process
- ⁶ pd complete process via log to expo form

- ⁴ $\log_3 x + \frac{1}{2} \log_3 x = 12$
- ⁵ $\log_3 x = 8$
- ⁶ $x = 3^8$ (= 6561)

or

- ⁴ $Q + \frac{1}{2}Q = 12$
- $Q = 8$
- ⁵ $\log_3 x = 8$
- ⁶ $x = 3^8$ (= 6561)

- ⁴ $2\log_9 x + \log_9 x = 12$
- ⁵ $\log_9 x = 4$
- ⁶ $x = 9^4$ (= 6561)

or

- ⁴ $2Q + Q = 12$
- $Q = 4$
- ⁵ $\log_9 x = 4$
- ⁶ $x = 9^4$ (= 6561)

Notes

2. At •⁴ any letter except x may be used in lieu of Q .
3. Candidates who use a trial and improvement technique by substituting values for x gain no marks.
4. The answer with no working gains no marks.

Regularly occurring responses

Response 1

$$\begin{aligned} Q + 2Q &= 12 \\ 3Q &= 12 \\ Q &= 4 \\ \log_3 x &= 4 \\ x &= 3^4 \\ &= 81 \end{aligned}$$

$$\begin{array}{|l} \checkmark \bullet^4 \\ \times \bullet^5 \end{array}$$

$$\begin{array}{|l} \times \bullet^4 \\ \checkmark \bullet^5 \end{array}$$

2 marks out of 3

Response 2

$$\begin{aligned} \log_3 x + 2\log_3 x &= 12 \quad \times \bullet^4 \\ 3\log_3 x &= 12 \\ \log_3 x &= 4 \quad \checkmark \bullet^5 \\ x &= 3^4 \quad \checkmark \bullet^6 \\ &= 81 \end{aligned}$$

2 marks out of 3

The marks allocated are dependent on what substitution is used for Q .

Response 3

$$\begin{aligned} 2\log_9 x + \log_9 x &= 12 \quad \checkmark \bullet^4 \\ \log_9 x^2 + \log_9 x &= 12 \\ \log_9 x^3 &= 12 \quad \checkmark \bullet^5 \\ x^3 &= 9^{12} \\ x &= \sqrt[3]{9^{12}} \\ x &= 9^4 \quad \checkmark \bullet^6 \\ x &= 3^8 \\ &= 6561 \end{aligned}$$

3 marks out of 3

Response 4

$$\begin{aligned} \log_3 x &= 8 \quad \times \bullet^4 \quad \times \bullet^5 \\ x &= 3^8 \quad \checkmark \bullet^6 \\ &= 6561 \end{aligned}$$

Without justification, •⁴ and •⁵ are not available.