

X100/302

NATIONAL
QUALIFICATIONS
2008

TUESDAY, 20 MAY
10.50 AM – 12.00 NOON

MATHEMATICS
HIGHER
Paper 2

Read Carefully

- 1 Calculators may be used in this paper.
- 2 Full credit will be given only where the solution contains appropriate working.
- 3 Answers obtained by readings from scale drawings will not receive any credit.



FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x - a)^2 + (y - b)^2 = r^2$ represents a circle centre (a, b) and radius r .

Scalar Product: $\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}| |\mathbf{b}| \cos \theta$, where θ is the angle between \mathbf{a} and \mathbf{b}

or $\mathbf{a} \cdot \mathbf{b} = a_1 b_1 + a_2 b_2 + a_3 b_3$ where $\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$.

Trigonometric formulae: $\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A$$

$$= 2 \cos^2 A - 1$$

$$= 1 - 2 \sin^2 A$$

Table of standard derivatives:

$f(x)$	$f'(x)$
$\sin ax$	$a \cos ax$
$\cos ax$	$-a \sin ax$

Table of standard integrals:

$f(x)$	$\int f(x) dx$
$\sin ax$	$-\frac{1}{a} \cos ax + C$
$\cos ax$	$\frac{1}{a} \sin ax + C$

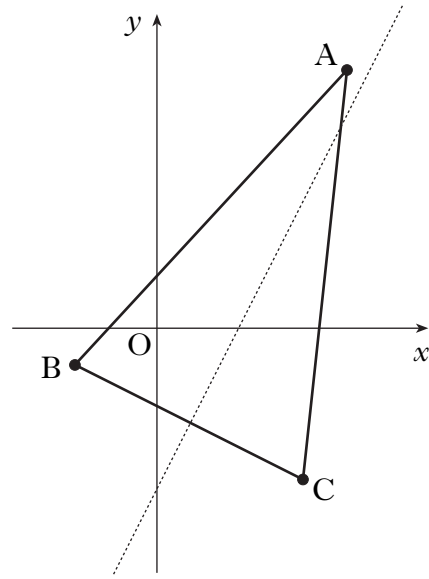
ALL questions should be attempted.

Marks

1. The vertices of triangle ABC are A(7, 9), B(-3, -1) and C(5, -5) as shown in the diagram.

The broken line represents the perpendicular bisector of BC.

- (a) Show that the equation of the perpendicular bisector of BC is $y = 2x - 5$.
- (b) Find the equation of the median from C.
- (c) Find the coordinates of the point of intersection of the perpendicular bisector of BC and the median from C.



4

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3

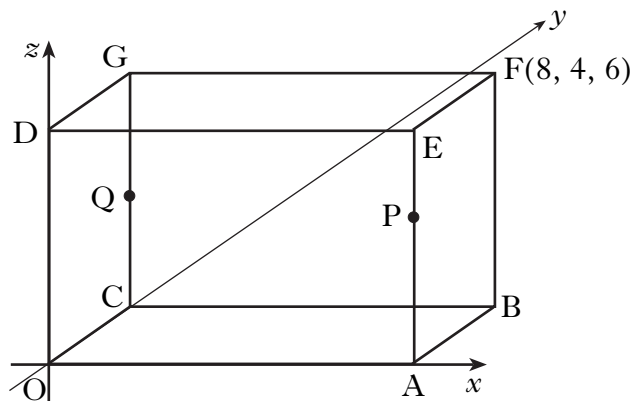
2. The diagram shows a cuboid OABC, DEFG.

F is the point (8, 4, 6).

P divides AE in the ratio 2:1.

Q is the midpoint of CG.

- (a) State the coordinates of P and Q.
- (b) Write down the components of \vec{PQ} and \vec{PA} .
- (c) Find the size of angle QPA.



2

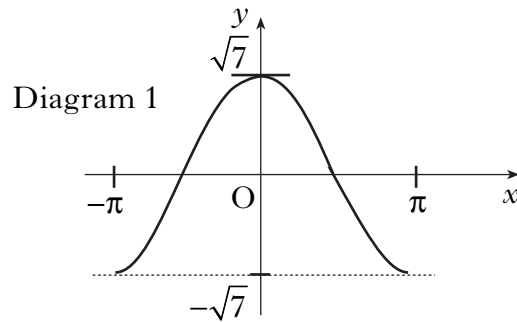
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5

[Turn over

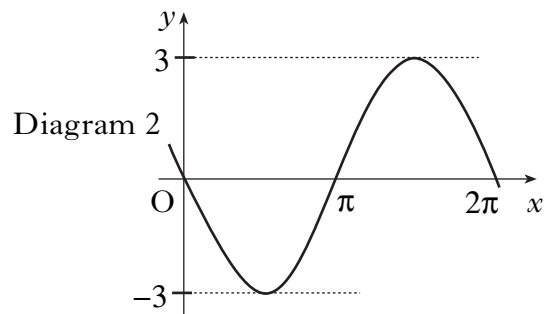
3. (a) (i) Diagram 1 shows part of the graph of $y = f(x)$, where $f(x) = p \cos x$.

Write down the value of p .



- (ii) Diagram 2 shows part of the graph of $y = g(x)$, where $g(x) = q \sin x$.

Write down the value of q .



- (b) Write $f(x) + g(x)$ in the form $k \cos(x + a)$ where $k > 0$ and $0 < a < \frac{\pi}{2}$. 4
- (c) Hence find $f'(x) + g'(x)$ as a single trigonometric expression. 2

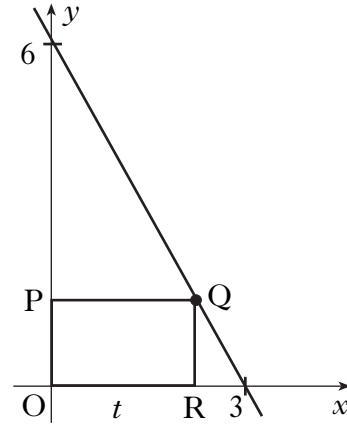
4. (a) Write down the centre and calculate the radius of the circle with equation $x^2 + y^2 + 8x + 4y - 38 = 0$. 2

- (b) A second circle has equation $(x - 4)^2 + (y - 6)^2 = 26$.
Find the distance between the centres of these two circles and hence show that the circles intersect. 4

- (c) The line with equation $y = 4 - x$ is a common chord passing through the points of intersection of the two circles.
Find the coordinates of the points of intersection of the two circles. 5

5. Solve the equation $\cos 2x^\circ + 2\sin x^\circ = \sin^2 x^\circ$ in the interval $0 \leq x < 360$. 5

6. In the diagram, Q lies on the line joining (0, 6) and (3, 0).
 OPQR is a rectangle, where P and R lie on the axes and $OR = t$.
- (a) Show that $QR = 6 - 2t$.
- (b) Find the coordinates of Q for which the rectangle has a maximum area.



3

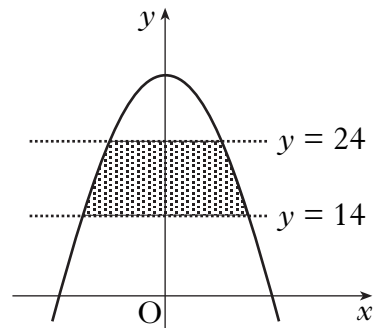
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7. The parabola shown in the diagram has equation

$$y = 32 - 2x^2.$$

The shaded area lies between the lines $y = 14$ and $y = 24$.

Calculate the shaded area.



8

[END OF QUESTION PAPER]