X100/301

NATIONAL QUALIFICATIONS 2008

TUESDAY, 20 MAY 9.00 AM – 10.30 AM

MATHEMATICS HIGHER
Paper 1
(Non-calculator)

Read carefully

Calculators may NOT be used in this paper.

Section A – Questions 1–20 (40 marks)
Instructions for completion of **Section A** are given on page two.
For this section of the examination you must use an **HB pencil**.

Section B (30 marks)

1 Full credit will be given only where the solution contains appropriate working.

2 Answers obtained by readings from scale drawings will not receive any credit.

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Read carefully

1 Check that the answer sheet provided is for **Mathematics Higher (Section A)**.

2 For this section of the examination you must use an **HB pencil** and, where necessary, an eraser.

3 Check that the answer sheet you have been given has **your name**, **date of birth**, **SCN** (Scottish Candidate Number) and **Centre Name** printed on it.

Do not change any of these details.

4 If any of this information is wrong, tell the Invigilator immediately.

5 If this information is correct, **print** your name and seat number in the boxes provided.

6 The answer to each question is **either** A, B, C or D. Decide what your answer is, then, using your pencil, put a horizontal line in the space provided (see sample question below).

7 There is **only one correct** answer to each question.

8 Rough working should **not** be done on your answer sheet.

9 At the end of the exam, put the **answer sheet for Section A inside the front cover of your answer book**.

Sample Question

A curve has equation $y = x^3 - 4x$.

What is the gradient at the point where $x = 2$?

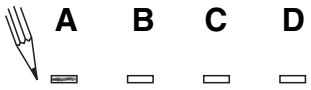
A 8

B 1

C 0

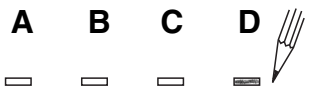
D −4

The correct answer is **A—**8. The answer **A** has been clearly marked in **pencil** with a horizontal line (see below).



Changing an answer

If you decide to change your answer, carefully erase your first answer and using your pencil, fill in the answer you want. The answer below has been changed to **D**.



FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x - a)^2 + (y - b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Scalar Product: $\boldsymbol{a}.\boldsymbol{b} = |\boldsymbol{a}|\,|\boldsymbol{b}| \cos\theta$, where θ is the angle between $\boldsymbol{a}$ and $\boldsymbol{b}$

or $\boldsymbol{a}.\boldsymbol{b} = a_1b_1 + a_2b_2 + a_3b_3$ where $\boldsymbol{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$.

Trigonometric formulae:

$$\begin{aligned} \sin(A \pm B) &= \sin A \cos B \pm \cos A \sin B \\ \cos(A \pm B) &= \cos A \cos B \mp \sin A \sin B \\ \sin 2A &= 2\sin A \cos A \\ \cos 2A &= \cos^2 A - \sin^2 A \\ &= 2\cos^2 A - 1 \\ &= 1 - 2\sin^2 A \end{aligned}$$

Table of standard derivatives:

$f(x)$	$f'(x)$
$\sin ax$	$a\cos ax$
$\cos ax$	$-a\sin ax$

Table of standard integrals:

$f(x)$	$\int f(x)\,dx$
$\sin ax$	$-\frac{1}{a}\cos ax + C$
$\cos ax$	$\frac{1}{a}\sin ax + C$

[Turn over

SECTION A

ALL questions should be attempted.

1. A sequence is defined by the recurrence relation

$$u_{n+1} = 0{\cdot}3u_n + 6 \text{ with } u_{10} = 10.$$

What is the value of u_{12}?

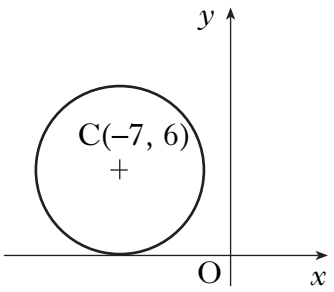
A 6·6

B 7·8

C 8·7

D 9·6

2. The x-axis is a tangent to a circle with centre (–7, 6) as shown in the diagram.



What is the equation of the circle?

A $(x + 7)^2 + (y - 6)^2 = 1$

B $(x + 7)^2 + (y - 6)^2 = 49$

C $(x - 7)^2 + (y + 6)^2 = 36$

D $(x + 7)^2 + (y - 6)^2 = 36$

3. The vectors $\boldsymbol{u} = \begin{pmatrix} k \\ -1 \\ 1 \end{pmatrix}$ and $\boldsymbol{v} = \begin{pmatrix} 0 \\ 4 \\ k \end{pmatrix}$ are perpendicular.

What is the value of k?

A 0

B 3

C 4

D 5

4. A sequence is generated by the recurrence relation $u_{n+1} = 0{\cdot}4u_n - 240$.

What is the limit of this sequence as $n \to \infty$?

A -800

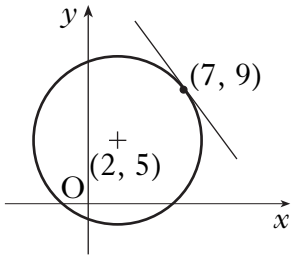
B -400

C 200

D 400

5. The diagram shows a circle, centre (2, 5) and a tangent drawn at the point (7, 9).

What is the equation of this tangent?



A $y - 9 = -\frac{5}{4}(x - 7)$

B $y + 9 = -\frac{4}{5}(x + 7)$

C $y - 7 = \frac{4}{5}(x - 9)$

D $y + 9 = \frac{5}{4}(x + 7)$

[Turn over

6. What is the solution of the equation $2\sin x - \sqrt{3} = 0$ where $\frac{\pi}{2} \leq x \leq \pi$?

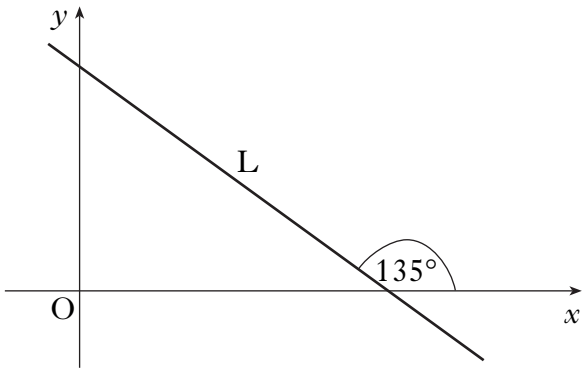
A $\frac{\pi}{6}$

B $\frac{2\pi}{3}$

C $\frac{3\pi}{4}$

D $\frac{5\pi}{6}$

7. The diagram shows a line L; the angle between L and the positive direction of the x-axis is 135°, as shown.



What is the gradient of line L?

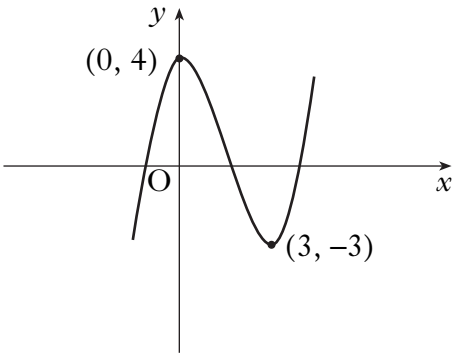
A $-\frac{1}{2}$

B $-\frac{\sqrt{3}}{2}$

C -1

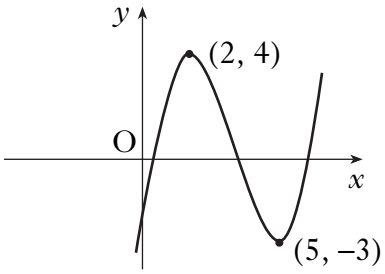
D $\frac{1}{2}$

8. The diagram shows part of the graph of a function with equation $y = f(x)$.



Which of the following diagrams shows the graph with equation $y = -f(x - 2)$?

A



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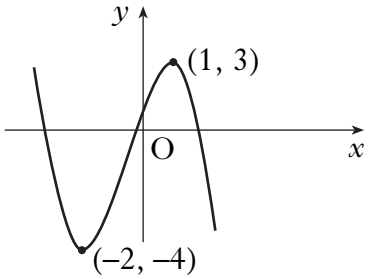
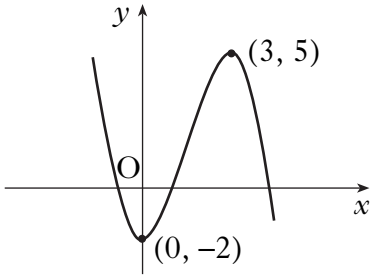
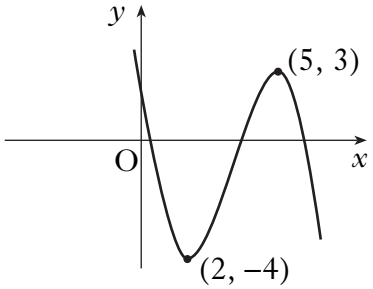


C



D



9. Given that $0 \leq a \leq \frac{\pi}{2}$ and $\sin a = \frac{3}{5}$, find an expression for $\sin(x + a)$.

A $\sin x + \frac{3}{5}$

B $\frac{4}{5}\sin x + \frac{3}{5}\cos x$

C $\frac{3}{5}\sin x - \frac{4}{5}\cos x$

D $\frac{2}{5}\sin x - \frac{3}{5}\cos x$

10. Here are two statements about the roots of the equation $x^2 + x + 1 = 0$:

(1) the roots are equal;

(2) the roots are real.

Which of the following is true?

A Neither statement is correct.

B Only statement (1) is correct.

C Only statement (2) is correct.

D Both statements are correct.

11. E(–2, –1, 4), P(1, 5, 7) and F(7, 17, 13) are three collinear points.

P lies between E and F.

What is the ratio in which P divides EF?

A 1:1

B 1:2

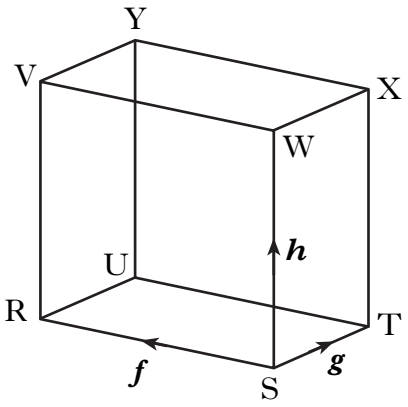
C 1:4

D 1:6

12. In the diagram RSTU, VWXY represents a cuboid.

$\overrightarrow{SR}$ represents vector $\boldsymbol{f}$, $\overrightarrow{ST}$ represents vector $\boldsymbol{g}$ and $\overrightarrow{SW}$ represents vector $\boldsymbol{h}$.

Express $\overrightarrow{VT}$ in terms of $\boldsymbol{f}$, $\boldsymbol{g}$ and $\boldsymbol{h}$.



A $\overrightarrow{VT} = \boldsymbol{f} + \boldsymbol{g} + \boldsymbol{h}$

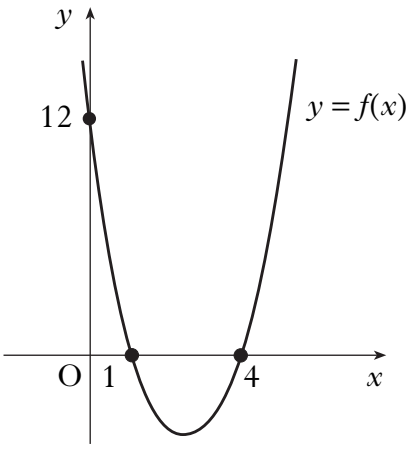
B $\overrightarrow{VT} = \boldsymbol{f} - \boldsymbol{g} + \boldsymbol{h}$

C $\overrightarrow{VT} = -\boldsymbol{f} + \boldsymbol{g} - \boldsymbol{h}$

D $\overrightarrow{VT} = -\boldsymbol{f} - \boldsymbol{g} + \boldsymbol{h}$

13. The diagram shows part of the graph of a quadratic function $y = f(x)$.

The graph has an equation of the form $y = k(x - a)(x - b)$.



What is the equation of the graph?

A $y = 3(x - 1)(x - 4)$

B $y = 3(x + 1)(x + 4)$

C $y = 12(x - 1)(x - 4)$

D $y = 12(x + 1)(x + 4)$

14. Find $\int 4\sin(2x + 3)\, dx$.

A $-4\cos(2x + 3) + c$

B $-2\cos(2x + 3) + c$

C $4\cos(2x + 3) + c$

D $8\cos(2x + 3) + c$

15. What is the derivative of $(x^3 + 4)^2$?

A $(3x^2 + 4)^2$

B $\frac{1}{3}(x^3 + 4)^3$

C $6x^2(x^3 + 4)$

D $2(3x^2 + 4)^{-1}$

16. $2x^2 + 4x + 7$ is expressed in the form $2(x + p)^2 + q$.

What is the value of q?

A 5

B 7

C 9

D 11

17. A function f is given by $f(x) = \sqrt{9 - x^2}$.

What is a suitable domain of f?

A $x \geq 3$

B $x \leq 3$

C $-3 \leq x \leq 3$

D $-9 \leq x \leq 9$

18. Vectors $\boldsymbol{p}$ and $\boldsymbol{q}$ are such that $|\boldsymbol{p}| = 3$, $|\boldsymbol{q}| = 4$ and $\boldsymbol{p}.\boldsymbol{q} = 10$.

Find the value of $\boldsymbol{q}.(\boldsymbol{p} + \boldsymbol{q})$.

A 0

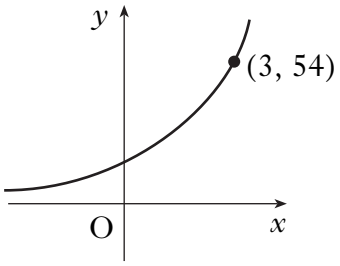
B 14

C 26

D 28

19. The diagram shows part of the graph whose equation is of the form $y = 2m^x$.

What is the value of m?



A 2

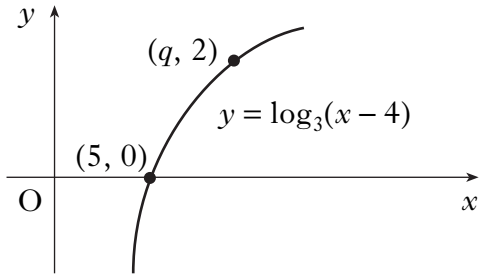
B 3

C 8

D 18

20. The diagram shows part of the graph of $y = \log_3(x - 4)$.

The point $(q, 2)$ lies on the graph.



What is the value of q?

A 6

B 7

C 8

D 13

[*END OF SECTION A*]

SECTION B

ALL questions should be attempted.

Marks

21. A function f is defined on the set of real numbers by $f(x) = x^3 - 3x + 2$.

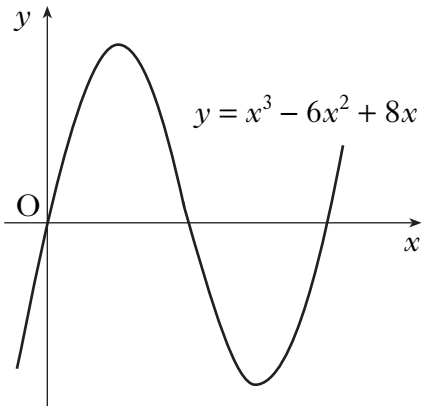
(*a*) Find the coordinates of the stationary points on the curve $y = f(x)$ and determine their nature. **6**

(*b*) (i) Show that $(x - 1)$ is a factor of $x^3 - 3x + 2$.

(ii) Hence or otherwise factorise $x^3 - 3x + 2$ fully. **5**

(*c*) State the coordinates of the points where the curve with equation $y = f(x)$ meets both the axes and hence sketch the curve. **4**

22. The diagram shows a sketch of the curve with equation $y = x^3 - 6x^2 + 8x$.



(*a*) Find the coordinates of the points on the curve where the gradient of the tangent is –1. **5**

(*b*) The line $y = 4 - x$ is a tangent to this curve at a point A. Find the coordinates of A. **2**

23. Functions f, g and h are defined on suitable domains by

$f(x) = x^2 - x + 10$, $g(x) = 5 - x$ and $h(x) = \log_2 x$.

(*a*) Find expressions for $h(f(x))$ and $h(g(x))$. **3**

(*b*) Hence solve $h(f(x)) - h(g(x)) = 3$. **5**

[*END OF SECTION B*]

[*END OF QUESTION PAPER*]

