

## FORMULAE LIST

### Circle:

The equation  $x^2 + y^2 + 2gx + 2fy + c = 0$  represents a circle centre  $(-g, -f)$  and radius  $\sqrt{g^2 + f^2 - c}$ .

The equation  $(x - a)^2 + (y - b)^2 = r^2$  represents a circle centre  $(a, b)$  and radius  $r$ .

**Scalar Product:**  $\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}| |\mathbf{b}| \cos \theta$ , where  $\theta$  is the angle between  $\mathbf{a}$  and  $\mathbf{b}$

or  $\mathbf{a} \cdot \mathbf{b} = a_1 b_1 + a_2 b_2 + a_3 b_3$  where  $\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$  and  $\mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$ .

**Trigonometric formulae:**  $\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A$$

$$= 2 \cos^2 A - 1$$

$$= 1 - 2 \sin^2 A$$

**Table of standard derivatives:**

$f(x)$	$f'(x)$
$\sin ax$	$a \cos ax$
$\cos ax$	$-a \sin ax$

**Table of standard integrals:**

$f(x)$	$\int f(x) dx$
$\sin ax$	$-\frac{1}{a} \cos ax + C$
$\cos ax$	$\frac{1}{a} \sin ax + C$

1. Find  $\int \frac{4x^3 - 1}{x^2} dx, x \neq 0.$

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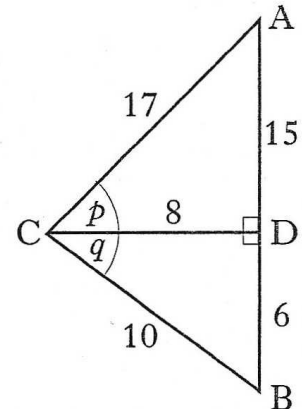
2. Triangles ACD and BCD are right-angled at D with angles  $p$  and  $q$  and lengths as shown in the diagram.

(a) Show that the exact value of  $\sin(p + q)$  is  $\frac{84}{85}.$

(b) Calculate the exact values of:

(i)  $\cos(p + q);$

(ii)  $\tan(p + q).$

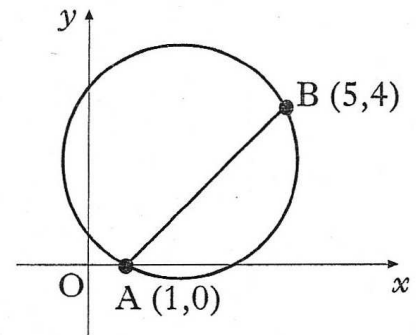


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3. (a) A chord joins the points A(1,0) and B(5,4) on the circle as shown in the diagram.

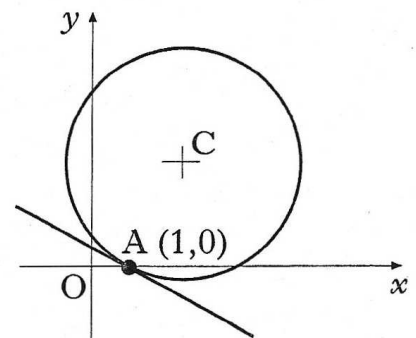
Show that the equation of the perpendicular bisector of chord AB is  $x + y = 5.$



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(b) The point C is the centre of this circle. The tangent at the point A on the circle has equation  $x + 3y = 1.$

Find the equation of the radius CA.



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(c) (i) Determine the coordinates of the point C.

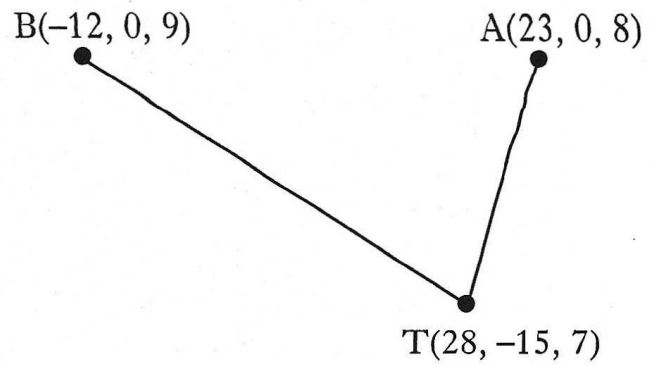
(ii) Find the equation of the circle.

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4. The sketch shows the positions of Andrew(A), Bob(B) and Tracy(T) on three hill-tops.

Relative to a suitable origin, the coordinates (in hundreds of metres) of the three people are A(23, 0, 8), B(-12, 0, 9) and T(28, -15, 7).

In the dark, Andrew and Bob locate Tracy using heat-seeking beams.



- (a) Express the vectors  $\vec{TA}$  and  $\vec{TB}$  in component form.

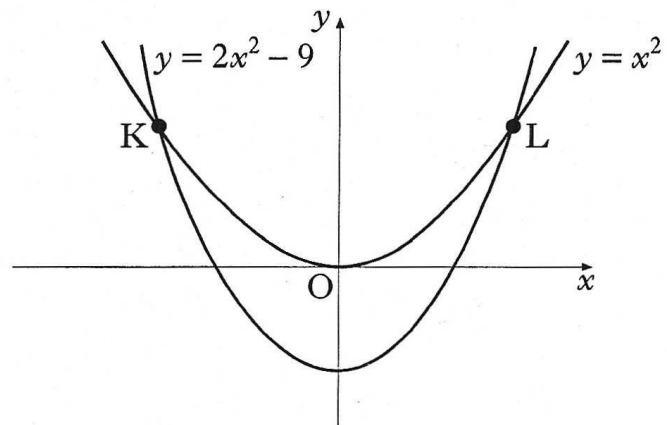
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- (b) Calculate the angle between these two beams.

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5. The curves with equations  $y = x^2$  and  $y = 2x^2 - 9$  intersect at K and L as shown.

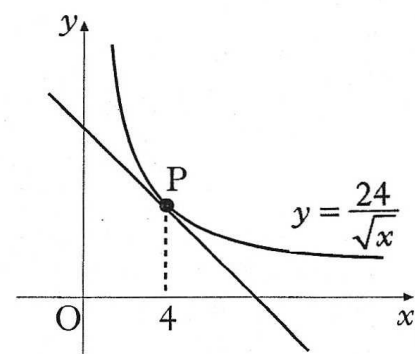
Calculate the area enclosed between the curves.



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6. The diagram shows the graph of  $y = \frac{24}{\sqrt{x}}$ ,  $x > 0$ .

Find the equation of the tangent at P, where  $x = 4$ .



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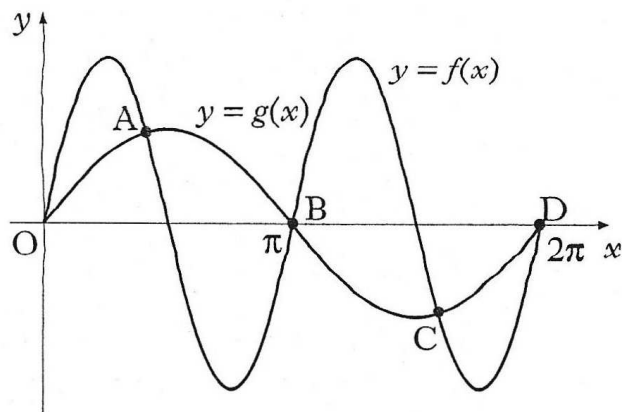
7. Solve the equation  $\log_4(5 - x) - \log_4(3 - x) = 2$ ,  $x < 3$ .

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8. Two functions,  $f$  and  $g$ , are defined by  $f(x) = k\sin 2x$  and  $g(x) = \sin x$  where  $k > 1$ .

The diagram shows the graphs of  $y = f(x)$  and  $y = g(x)$  intersecting at  $O, A, B, C$  and  $D$ .

Show that, at  $A$  and  $C$ ,  $\cos x = \frac{1}{2k}$ .



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9. The value  $V$  (in £ million) of a cruise ship  $t$  years after launch is given by the formula  $V = 252e^{-0.06335t}$ .

(a) What was its value when launched?

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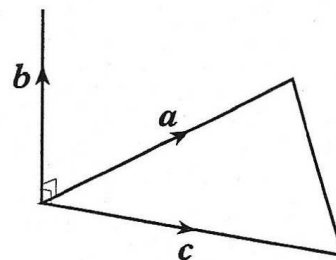
(b) The owners decide to sell the ship once its value falls below £20 million. After how many years will it be sold?

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10. Vectors  $\mathbf{a}$  and  $\mathbf{c}$  are represented by two sides of an equilateral triangle with sides of length 3 units, as shown in the diagram.

Vector  $\mathbf{b}$  is 2 units long and  $\mathbf{b}$  is perpendicular to both  $\mathbf{a}$  and  $\mathbf{c}$ .

Evaluate the scalar product  $\mathbf{a} \cdot (\mathbf{a} + \mathbf{b} + \mathbf{c})$ .



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11. (a) Show that  $x = -1$  is a solution of the cubic equation  $x^3 + px^2 + px + 1 = 0$ .

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(b) Hence find the range of values of  $p$  for which all the roots of the cubic equation are real.

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