

FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x - a)^2 + (y - b)^2 = r^2$ represents a circle centre (a, b) and radius r .

Scalar Product: $\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}| |\mathbf{b}| \cos \theta$, where θ is the angle between \mathbf{a} and \mathbf{b}

or $\mathbf{a} \cdot \mathbf{b} = a_1 b_1 + a_2 b_2 + a_3 b_3$ where $\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$.

Trigonometric formulae: $\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A$$

$$= 2 \cos^2 A - 1$$

$$= 1 - 2 \sin^2 A$$

Table of standard derivatives:

$f(x)$	$f'(x)$
$\sin ax$	$a \cos ax$
$\cos ax$	$-a \sin ax$

Table of standard integrals:

$f(x)$	$\int f(x) dx$
$\sin ax$	$-\frac{1}{a} \cos ax + C$
$\cos ax$	$\frac{1}{a} \sin ax + C$

1. $f(x) = 6x^3 - 5x^2 - 17x + 6$.

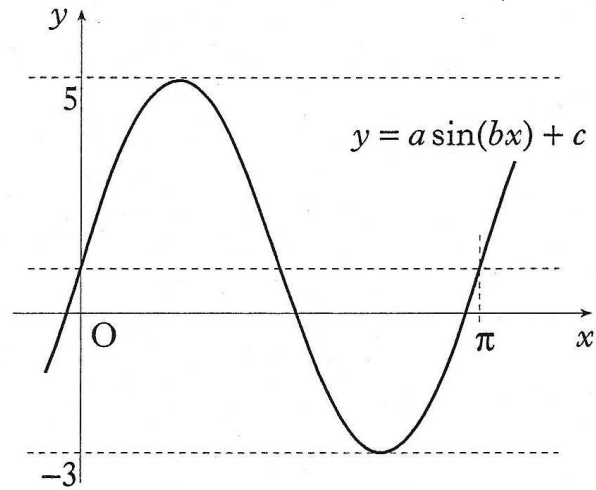
(a) Show that $(x - 2)$ is a factor of $f(x)$.

(b) Express $f(x)$ in its fully factorised form.

4

2. The diagram shows a sketch of part of the graph of a trigonometric function whose equation is of the form $y = a \sin(bx) + c$.

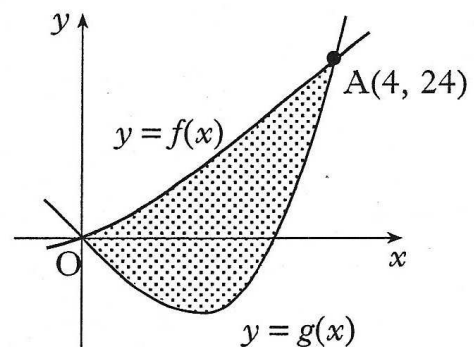
Determine the values of a , b and c .



3

3. The incomplete graphs of $f(x) = x^2 + 2x$ and $g(x) = x^3 - x^2 - 6x$ are shown in the diagram. The graphs intersect at $A(4, 24)$ and the origin.

Find the shaded area enclosed between the curves.



5

4. (a) Find the equation of the tangent to the curve with equation $y = x^3 + 2x^2 - 3x + 2$ at the point where $x = 1$.

5

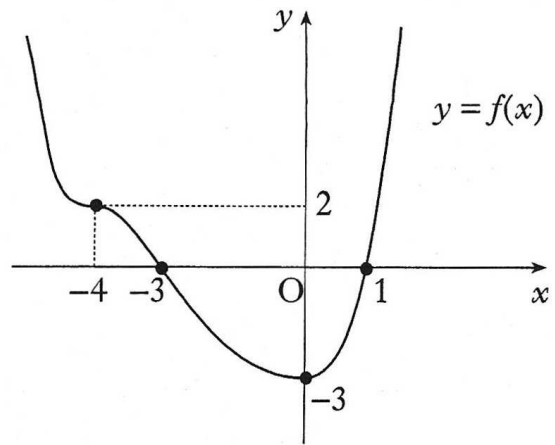
(b) Show that this line is also a tangent to the circle with equation $x^2 + y^2 - 12x - 10y + 44 = 0$ and state the coordinates of the point of contact.

6

5. The diagram shows the graph of a function f .

f has a minimum turning point at $(0, -3)$ and a point of inflexion at $(-4, 2)$.

- (a) Sketch the graph of $y = f(-x)$.
 (b) On the same diagram, sketch the graph of $y = 2f(-x)$.



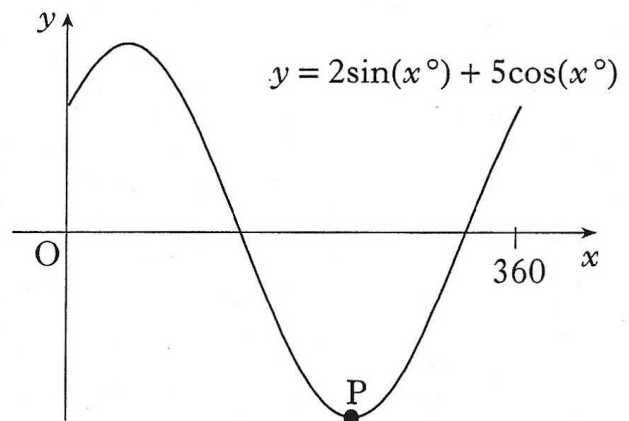
2
2

6. If $f(x) = \cos(2x) - 3 \sin(4x)$, find the exact value of $f'\left(\frac{\pi}{6}\right)$.

4

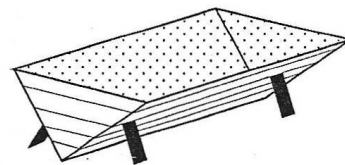
7. Part of the graph of $y = 2\sin(x^\circ) + 5\cos(x^\circ)$ is shown in the diagram.

- (a) Express $y = 2\sin(x^\circ) + 5\cos(x^\circ)$ in the form $k\sin(x^\circ + a^\circ)$ where $k > 0$ and $0 \leq a < 360$.
 (b) Find the coordinates of the minimum turning point P.

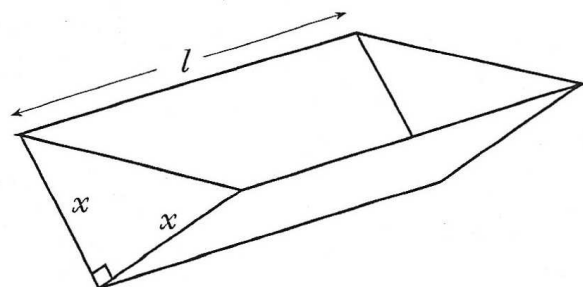


4
3

8. An open water tank, in the shape of a triangular prism, has a capacity of 108 litres. The tank is to be lined on the inside in order to make it watertight.



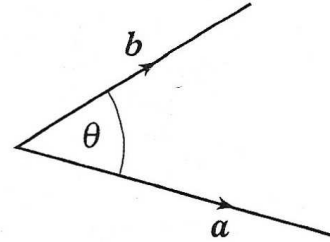
The triangular cross-section of the tank is right-angled and isosceles, with equal sides of length x cm. The tank has a length of l cm.



- (a) Show that the surface area to be lined, A cm², is given by $A(x) = x^2 + \frac{432000}{x}$.
 (b) Find the value of x which minimises this surface area.

3
5

9. The diagram shows vectors \mathbf{a} and \mathbf{b} .
If $|\mathbf{a}| = 5$, $|\mathbf{b}| = 4$ and $\mathbf{a} \cdot (\mathbf{a} + \mathbf{b}) = 36$, find the size of the acute angle θ between \mathbf{a} and \mathbf{b} .



10. Solve the equation $3\cos(2x) + 10\cos(x) - 1 = 0$ for $0 \leq x \leq \pi$, correct to 2 decimal places.
11. (a) (i) Sketch the graph of $y = a^x + 1$, $a > 2$.
(ii) On the same diagram, sketch the graph of $y = a^{x+1}$, $a > 2$.
(b) Prove that the graphs intersect at a point where the x -coordinate is $\log_a\left(\frac{1}{a-1}\right)$.