

# X100/301

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NATIONAL  
QUALIFICATIONS  
2002

MONDAY, 27 MAY  
9.00 AM – 10.10 AM

MATHEMATICS  
HIGHER

Units 1, 2 and 3

Paper 1

(Non-calculator)

## Read Carefully

- 1 Calculators may **NOT** be used in this paper.
- 2 Full credit will be given only where the solution contains appropriate working.
- 3 Answers obtained by readings from scale drawings will not receive any credit.



## FORMULAE LIST

### Circle:

The equation  $x^2 + y^2 + 2gx + 2fy + c = 0$  represents a circle centre  $(-g, -f)$  and radius  $\sqrt{g^2 + f^2 - c}$ .

The equation  $(x - a)^2 + (y - b)^2 = r^2$  represents a circle centre  $(a, b)$  and radius  $r$ .

**Scalar Product:**  $\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}| |\mathbf{b}| \cos \theta$ , where  $\theta$  is the angle between  $\mathbf{a}$  and  $\mathbf{b}$

or  $\mathbf{a} \cdot \mathbf{b} = a_1 b_1 + a_2 b_2 + a_3 b_3$  where  $\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$  and  $\mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$ .

**Trigonometric formulae:**  $\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A$$

$$= 2 \cos^2 A - 1$$

$$= 1 - 2 \sin^2 A$$

**Table of standard derivatives:**

$f(x)$	$f'(x)$
$\sin ax$	$a \cos ax$
$\cos ax$	$-a \sin ax$

**Table of standard integrals:**

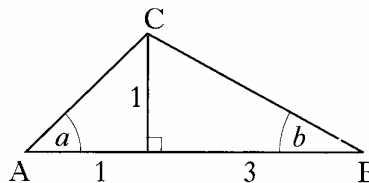
$f(x)$	$\int f(x) dx$
$\sin ax$	$-\frac{1}{a} \cos ax + C$
$\cos ax$	$\frac{1}{a} \sin ax + C$

ALL questions should be attempted.

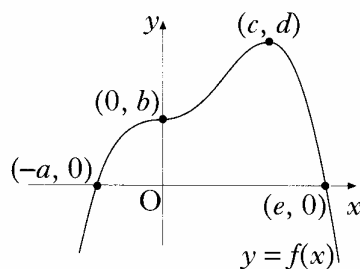
Marks

1. The point  $P(2, 3)$  lies on the circle  $(x + 1)^2 + (y - 1)^2 = 13$ . Find the equation of the tangent at  $P$ . 4
2. The point  $Q$  divides the line joining  $P(-1, -1, 0)$  to  $R(5, 2, -3)$  in the ratio  $2 : 1$ . Find the coordinates of  $Q$ . 3
3. Functions  $f$  and  $g$  are defined on suitable domains by  $f(x) = \sin(x^\circ)$  and  $g(x) = 2x$ .
- (a) Find expressions for:
- (i)  $f(g(x))$ ;
- (ii)  $g(f(x))$ . 2
- (b) Solve  $2f(g(x)) = g(f(x))$  for  $0 \leq x \leq 360$ . 5
4. Find the coordinates of the point on the curve  $y = 2x^2 - 7x + 10$  where the tangent to the curve makes an angle of  $45^\circ$  with the positive direction of the  $x$ -axis. 4

5. In triangle  $ABC$ , show that the exact value of  $\sin(a + b)$  is  $\frac{2}{\sqrt{5}}$ . 4



6. The graph of a function  $f$  intersects the  $x$ -axis at  $(-a, 0)$  and  $(e, 0)$  as shown. There is a point of inflexion at  $(0, b)$  and a maximum turning point at  $(c, d)$ . Sketch the graph of the derived function  $f'$ . 3

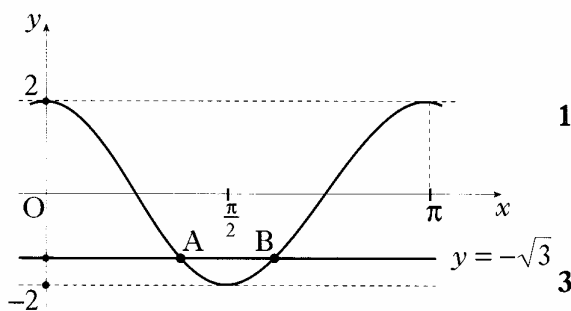


[Turn over for Questions 7 to 11 on Page four

7. (a) Express  $f(x) = x^2 - 4x + 5$  in the form  $f(x) = (x - a)^2 + b$ . 2
- (b) On the same diagram sketch:
- (i) the graph of  $y = f(x)$ ;
- (ii) the graph of  $y = 10 - f(x)$ . 4
- (c) Find the range of values of  $x$  for which  $10 - f(x)$  is positive. 1

8. The diagram shows the graph of a cosine function from 0 to  $\pi$ .

- (a) State the equation of the graph. 1
- (b) The line with equation  $y = -\sqrt{3}$  intersects this graph at points A and B. 3
- Find the coordinates of B.

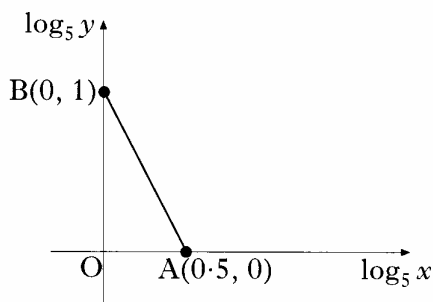


9. (a) Write  $\sin(x) - \cos(x)$  in the form  $k\sin(x - a)$  stating the values of  $k$  and  $a$  where  $k > 0$  and  $0 \leq a \leq 2\pi$ . 4
- (b) Sketch the graph of  $y = \sin(x) - \cos(x)$  for  $0 \leq x \leq 2\pi$ , showing clearly the graph's maximum and minimum values and where it cuts the  $x$ -axis and the  $y$ -axis. 3

10. (a) Find the derivative of the function  $f(x) = (8 - x^3)^{\frac{1}{2}}$ ,  $x < 2$ . 2

- (b) Hence write down  $\int \frac{x^2}{(8 - x^3)^{\frac{1}{2}}} dx$ . 1

11. The graph illustrates the law  $y = kx^n$ . If the straight line passes through A(0.5, 0) and B(0, 1), find the values of  $k$  and  $n$ . 4



[END OF QUESTION PAPER]