

X056/301

NATIONAL
QUALIFICATIONS
2001

THURSDAY, 17 MAY
9.00 AM - 10.10 AM

MATHEMATICS
HIGHER

Units 1, 2 and 3

Paper 1

(Non-calculator)

Read Carefully

- 1 Calculators may **NOT** be used in this paper.
- 2 Full credit will be given only where the solution contains appropriate working.
- 3 Answers obtained by readings from scale drawings will not receive any credit.

FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x - a)^2 + (y - b)^2 = r^2$ represents a circle centre (a, b) and radius r .

Scalar Product: $\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}| |\mathbf{b}| \cos \theta$, where θ is the angle between \mathbf{a} and \mathbf{b}

or $\mathbf{a} \cdot \mathbf{b} = a_1 b_1 + a_2 b_2 + a_3 b_3$ where $\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$.

Trigonometric formulae: $\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A$$

$$= 2 \cos^2 A - 1$$

$$= 1 - 2 \sin^2 A$$

Table of standard derivatives:

$f(x)$	$f'(x)$
$\sin ax$	$a \cos ax$
$\cos ax$	$-a \sin ax$

Table of standard integrals:

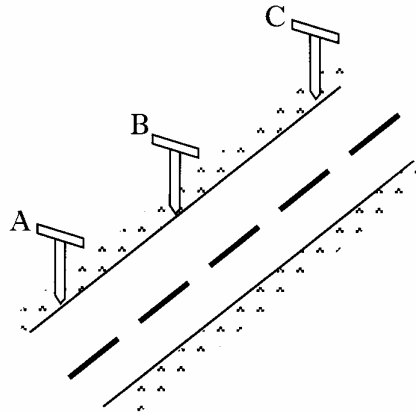
$f(x)$	$\int f(x) dx$
$\sin ax$	$-\frac{1}{a} \cos ax + C$
$\cos ax$	$\frac{1}{a} \sin ax + C$

ALL questions should be attempted.

Marks

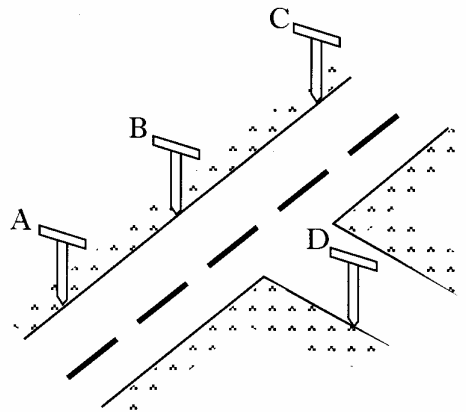
1. Find the equation of the straight line which is parallel to the line with equation $2x + 3y = 5$ and which passes through the point $(2, -1)$. 3
2. For what value of k does the equation $x^2 - 5x + (k + 6) = 0$ have equal roots? 3

3. (a) Roadmakers look along the tops of a set of T-rods to ensure that straight sections of road are being created. Relative to suitable axes the top left corners of the T-rods are the points $A(-8, -10, -2)$, $B(-2, -1, 1)$ and $C(6, 11, 5)$.
Determine whether or not the section of road ABC has been built in a straight line.



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- (b) A further T-rod is placed such that D has coordinates $(1, -4, 4)$.
Show that DB is perpendicular to AB.



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4. Given $f(x) = x^2 + 2x - 8$, express $f(x)$ in the form $(x + a)^2 - b$. 2

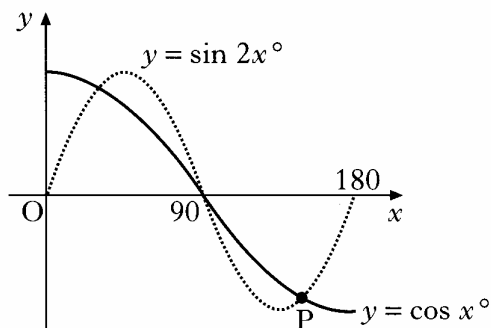
[Turn over

5. (a) Solve the equation $\sin 2x^\circ - \cos x^\circ = 0$ in the interval $0 \leq x \leq 180$.

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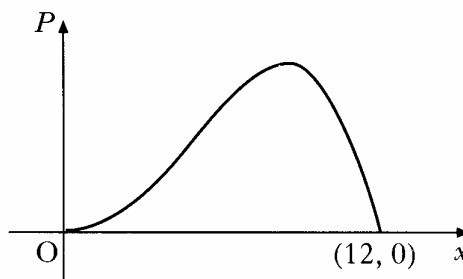
- (b) The diagram shows parts of two trigonometric graphs, $y = \sin 2x^\circ$ and $y = \cos x^\circ$.

Use your solutions in (a) to write down the coordinates of the point P.



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6. A company spends x thousand pounds a year on advertising and this results in a profit of P thousand pounds. A mathematical model, illustrated in the diagram, suggests that P and x are related by $P = 12x^3 - x^4$ for $0 \leq x \leq 12$. Find the value of x which gives the maximum profit.



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7. Functions $f(x) = \sin x$, $g(x) = \cos x$ and $h(x) = x + \frac{\pi}{4}$ are defined on a suitable set of real numbers.

(a) Find expressions for:

- (i) $f(h(x))$;
 (ii) $g(h(x))$.

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(b) (i) Show that $f(h(x)) = \frac{1}{\sqrt{2}} \sin x + \frac{1}{\sqrt{2}} \cos x$.

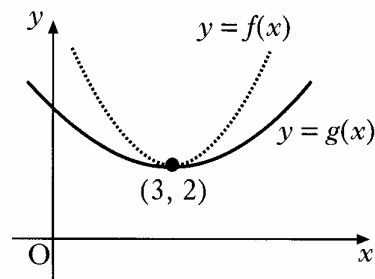
- (ii) Find a similar expression for $g(h(x))$ and hence solve the equation $f(h(x)) - g(h(x)) = 1$ for $0 \leq x \leq 2\pi$.

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8. Find x if $4 \log_x 6 - 2 \log_x 4 = 1$.

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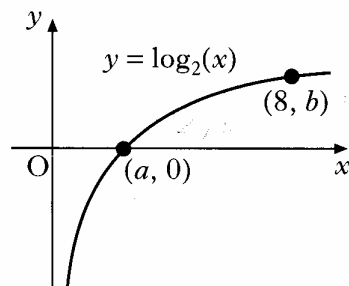
9. The diagram shows the graphs of two quadratic functions $y = f(x)$ and $y = g(x)$. Both graphs have a minimum turning point at $(3, 2)$.



Sketch the graph of $y = f'(x)$ and on the same diagram sketch the graph of $y = g'(x)$.

2

10. The diagram shows a sketch of part of the graph of $y = \log_2(x)$.



(a) State the values of a and b .

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(b) Sketch the graph of $y = \log_2(x + 1) - 3$.

3

11. Circle P has equation $x^2 + y^2 - 8x - 10y + 9 = 0$. Circle Q has centre $(-2, -1)$ and radius $2\sqrt{2}$.

(a) (i) Show that the radius of circle P is $4\sqrt{2}$.

(ii) Hence show that circles P and Q touch.

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(b) Find the equation of the tangent to circle Q at the point $(-4, 1)$.

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(c) The tangent in (b) intersects circle P in two points. Find the x -coordinates of the points of intersection, expressing your answers in the form $a \pm b\sqrt{3}$.

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[END OF QUESTION PAPER]