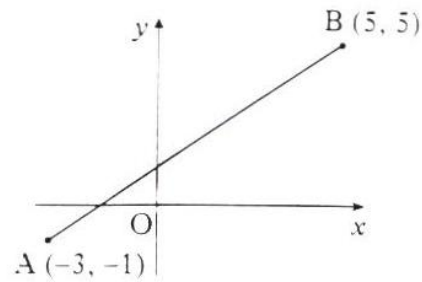


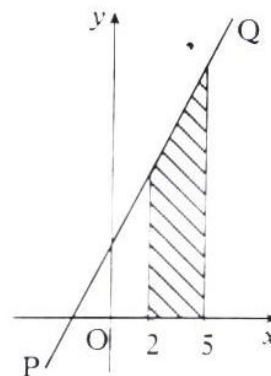
**All questions should be attempted***Marks*

1. (a) Show that  $x = 2$  is a root of the equation  $2x^3 + x^2 - 13x + 6 = 0$ . (1)  
 (b) Hence find the other roots. (3)

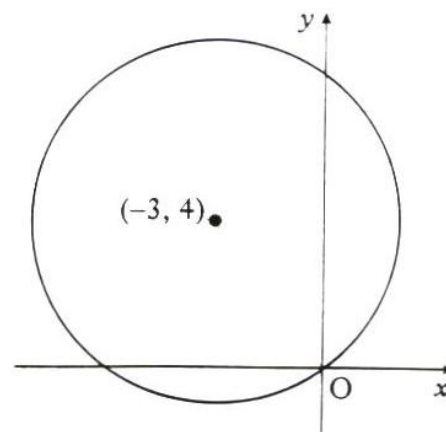
2. A and B are the points  $(-3, -1)$  and  $(5, 5)$ .  
 Find the equation of  
 (a) the line AB (2)  
 (b) the perpendicular bisector of AB. (3)



3. The line PQ has equation  $y = 2x + 4$ .  
 (a) Find, without using calculus, the area of the shaded trapezium shown in the diagram. (2)  
 (b) Express the area of this trapezium as a definite integral. (1)  
 (c) Evaluate this integral. (2)



4. Find the equation of the circle with centre  $(-3, 4)$  and passing through the origin. (2)



Marks

5. Given  $f(x) = 3x^2(2x - 1)$ , find  $f'(-1)$ .

(3)

6.  $VABCD$  is a pyramid with rectangular base  $ABCD$ .

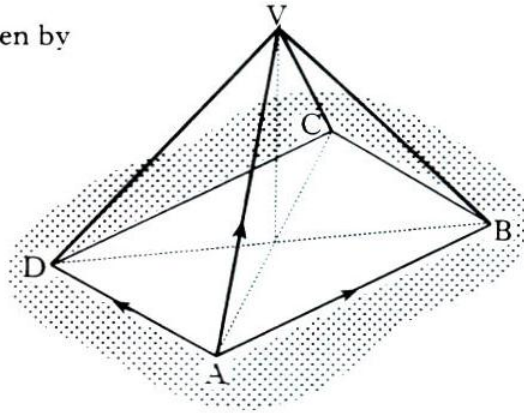
The vectors  $\vec{AB}$ ,  $\vec{AD}$  and  $\vec{AV}$  are given by

$$\vec{AB} = 8\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$$

$$\vec{AD} = -2\mathbf{i} + 10\mathbf{j} - 2\mathbf{k} \quad \text{and}$$

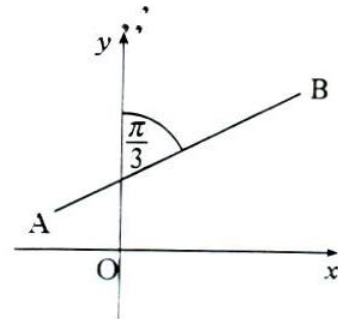
$$\vec{AV} = \mathbf{i} + 7\mathbf{j} + 7\mathbf{k}.$$

Express  $\vec{CV}$  in component form.



(3)

7. The line  $AB$  makes an angle of  $\frac{\pi}{3}$  radians with the  $y$ -axis, as shown in the diagram. Find the exact value of the gradient of  $AB$ .



(2)

8. (i) Write down the condition for the equation  $ax^2 + bx + c = 0$  to have equal roots.  
 (ii) Hence, or otherwise, show that the equation  $x(x + 7) = x - 9$  has equal roots.

(3)

9. The point  $P(-1, 7)$  lies on the curve with equation  $y = 5x^2 + 2$ . Find the equation of the tangent to the curve at  $P$ .

(4)

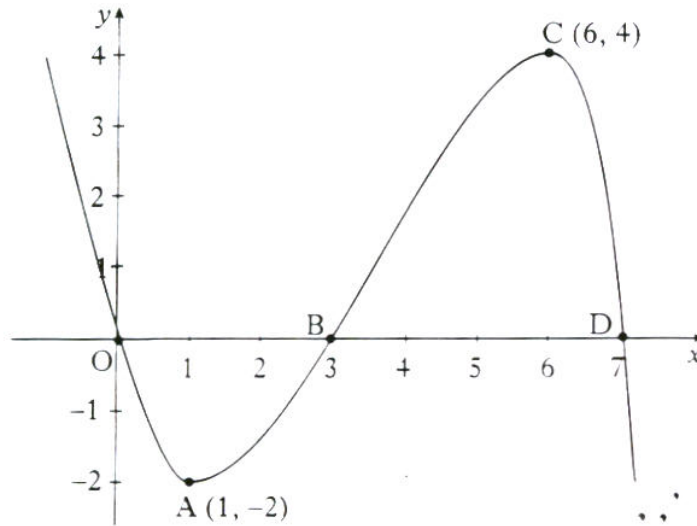
Marks

10. Part of the graph of  $y = f(x)$  is shown in the diagram. On separate diagrams, sketch the graph of

(a)  $y = f(x + 1)$  (2)

(b)  $y = -2f(x)$ . (3)

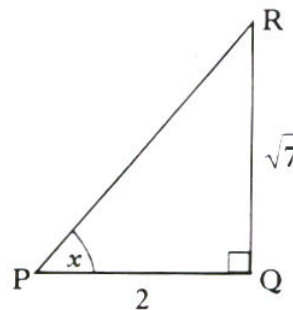
Indicate on each graph the images of O, A, B, C and D.



11. The graph of  $y = g(x)$  passes through the point  $(1, 2)$ .

If  $\frac{dy}{dx} = x^3 + \frac{1}{x^2} - \frac{1}{4}$ , express  $y$  in terms of  $x$ . (4)

12. Using triangle PQR, as shown, find the exact value of  $\cos 2x$ . (3)

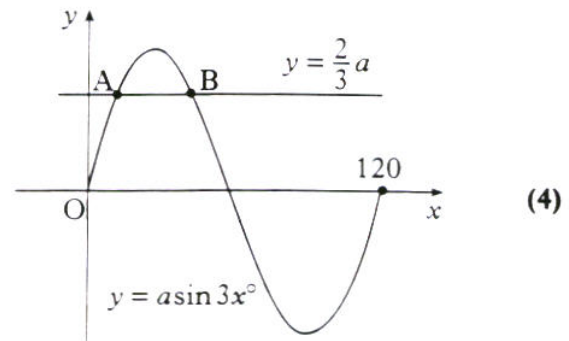


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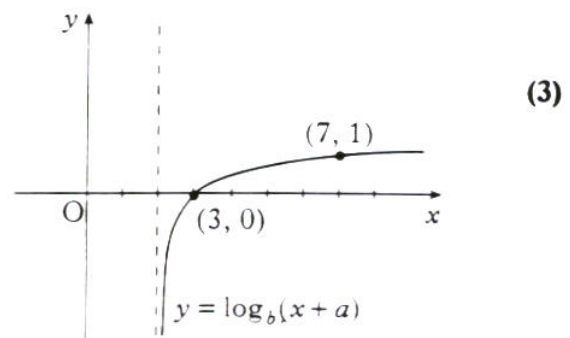
13. (a) Show that  $f(x) = 2x^2 - 4x + 5$  can be written in the form  $f(x) = a(x + b)^2 + c$ . (3)
- (b) Hence write down the coordinates of the stationary point of  $y = f(x)$  and state its nature. (2)

14. The diagram shows part of the graph of  $y = a \sin 3x^\circ$  and the line with equation  $y = \frac{2}{3}a$ .

Find the  $x$ -coordinates of A and B.



15. The diagram shows part of the graph of  $y = \log_b(x + a)$ .
- Determine the values of  $a$  and  $b$ .

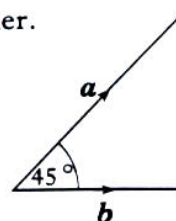


16. A curve has equation  $y = 2x^3 + 3x^2 + 4x - 5$ .
- Prove that this curve has no stationary points. (5)

17. The diagram shows two vectors  $\mathbf{a}$  and  $\mathbf{b}$ , with  $|\mathbf{a}| = 3$  and  $|\mathbf{b}| = 2\sqrt{2}$ . Higher Past Paper - 1999 - P1

These vectors are inclined at an angle of  $45^\circ$  to each other.

- (a) Evaluate (i)  $\mathbf{a} \cdot \mathbf{a}$   
 (ii)  $\mathbf{b} \cdot \mathbf{b}$   
 (iii)  $\mathbf{a} \cdot \mathbf{b}$



(2)

(b) Another vector  $\mathbf{p}$  is defined by  $\mathbf{p} = 2\mathbf{a} + 3\mathbf{b}$ .

Evaluate  $\mathbf{p} \cdot \mathbf{p}$  and hence write down  $|\mathbf{p}|$ .

(4)

18. Two sequences are defined by the recurrence relations

$$u_{n+1} = 0.2u_n + p, \quad u_0 = 1 \quad \text{and}$$

$$v_{n+1} = 0.6v_n + q, \quad v_0 = 1.$$

If both sequences have the same limit, express  $p$  in terms of  $q$ .

(3)

19. Given  $f(x) = \cos^2 x - \sin^2 x$ , find  $f'(x)$ .

(3)

20. Find  $\int \frac{x^2 - 5}{x\sqrt{x}} dx$ .

(4)

21. A function  $f$  can be expressed as an infinite series by

$$f(x) = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24} + \frac{x^5}{120} + \dots$$

(a) Write down the series for  $f(2x)$  as far as the term in  $x^5$ .

(1)

The derivative of  $f(x)$  can be calculated as follows.

$$f(x) = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24} + \frac{x^5}{120} + \dots$$

$$\text{so } f'(x) = 0 + 1 + \frac{2x}{2} + \frac{3x^2}{6} + \frac{4x^3}{24} + \frac{5x^4}{120} + \dots$$

$$= 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24} + \dots$$

$$\text{ie } f'(x) = f(x)$$

(b) Find  $f'(2x)$  in terms of  $f(2x)$ .

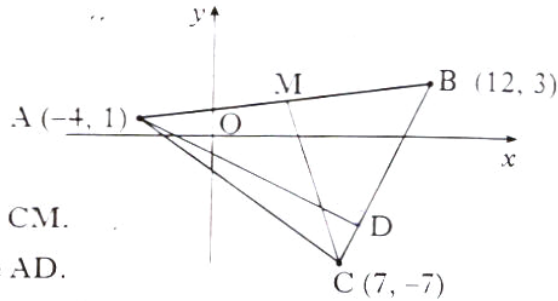
(3)

[END OF QUESTION PAPER]

All questions should be attempted

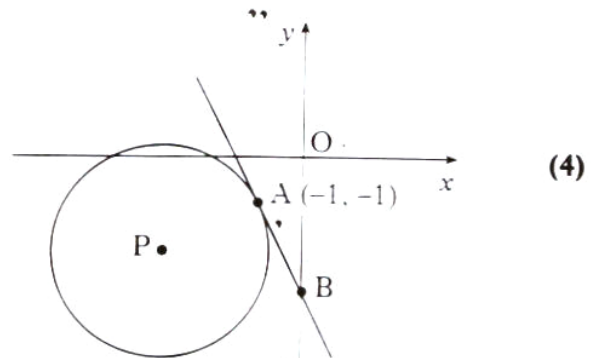
Marks

1. A triangle ABC has vertices  
A (-4, 1), B (12, 3) and C (7, -7).



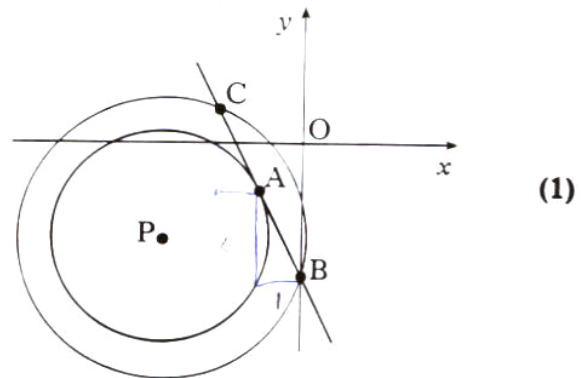
- (a) Find the equation of the median CM. (3)  
 (b) Find the equation of the altitude AD. (3)  
 (c) Find the coordinates of the point of intersection of CM and AD. (3)

2. (a) The diagram shows a circle, centre P, with equation  $x^2 + y^2 + 6x + 4y + 8 = 0$ .  
Find the equation of the tangent at the point A (-1, -1) on the circle.



- (b) The tangent crosses the y-axis at B.  
Find the coordinates of B. (1)

- (c) Another circle, centre P, is drawn passing through B. The tangent at A meets the second circle at the point C, as shown in the diagram.



- Write down the coordinates of C. (1)  
 (d) Find the equation of the circle with BC as diameter. (2)

Marks

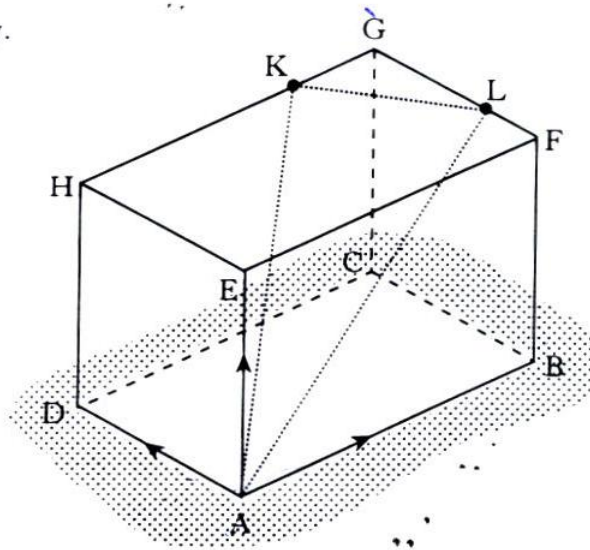
3. ABCDEFGH is a cuboid.

K lies two thirds of the way along HG, (ie  $HK:KG = 2:1$ ).

L lies one quarter of the way along FG, (ie  $FL:LG = 1:3$ ).

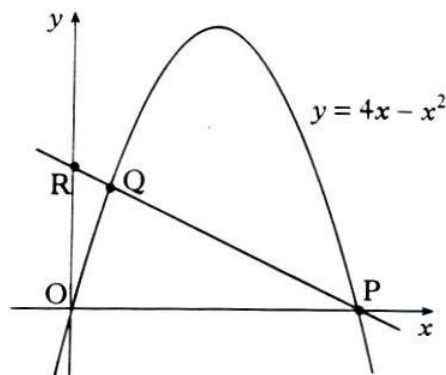
$\vec{AB}$ ,  $\vec{AD}$  and  $\vec{AE}$  can be represented by the vectors

$$\begin{pmatrix} 3 \\ 6 \\ 3 \end{pmatrix}, \begin{pmatrix} -8 \\ + \\ + \end{pmatrix} \text{ and } \begin{pmatrix} 1 \\ -3 \\ 5 \end{pmatrix} \text{ respectively.}$$



- (a) Calculate the components of  $\vec{AK}$ . (2)
- (b) Calculate the components of  $\vec{AL}$ . (2)
- (c) Calculate the size of angle KAL. (5)

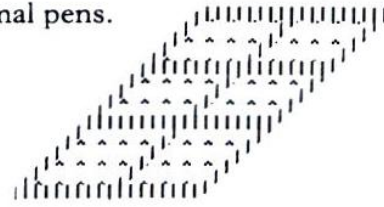
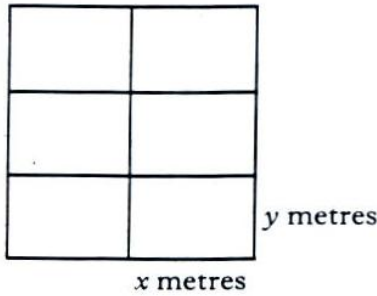
4. The parabola shown in the diagram has equation  $y = 4x - x^2$  and intersects the  $x$ -axis at the origin and P.



- (a) Find the coordinates of the point P. (2)
- (b) R is the point (0, 2). Find the equation of PR. (2)
- (c) The line and the parabola also intersect at Q. Find the coordinates of Q. (4)

Marks

5. A zookeeper wants to fence off six individual animal pens.



Each pen is a rectangle measuring  $x$  metres by  $y$  metres, as shown in the diagram.

- (a) (i) Express the total length of fencing in terms of  $x$  and  $y$ .  
 (ii) Given that the total length of fencing is 360 m, show that the total area,  $A \text{ m}^2$ , of the six pens is given by  $A(x) = 240x - \frac{16}{3}x^2$ . (4)
- (b) Find the values of  $x$  and  $y$  which give the maximum area and write down this maximum area. (6)

6. Functions  $f$  and  $g$  are defined on the set of real numbers by

$$f(x) = x - 1$$

$$g(x) = x^2.$$

- (a) Find formulae for  
 (i)  $f(g(x))$   
 (ii)  $g(f(x))$ . (4)
- (b) The function  $h$  is defined by  $h(x) = f(g(x)) + g(f(x))$ .  
 Show that  $h(x) = 2x^2 - 2x$  and sketch the graph of  $h$ . (3)
- (c) Find the area enclosed between this graph and the  $x$ -axis. (4)

[Turn over

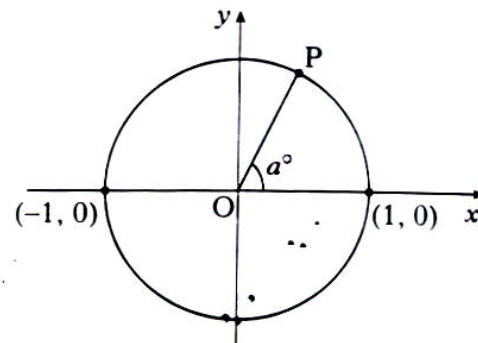
**Marks**

7. The intensity  $I_t$  of light is reduced as it passes through a filter according to the law  $I_t = I_0 e^{-kt}$  where  $I_0$  is the initial intensity and  $I_t$  is the intensity after passing through a filter of thickness  $t$  cm.  $k$  is a constant.

(a) A filter of thickness 4 cm reduces the intensity from 120 candle-power to 90 candle-power. Find the value of  $k$ . (4)

(b) Light is passed through a filter of thickness 10 cm. Find the percentage reduction in its intensity. (3)

8. The diagram shows a circle of radius 1 unit and centre the origin. The radius  $OP$  makes an angle  $a^\circ$  with the positive direction of the  $x$ -axis.



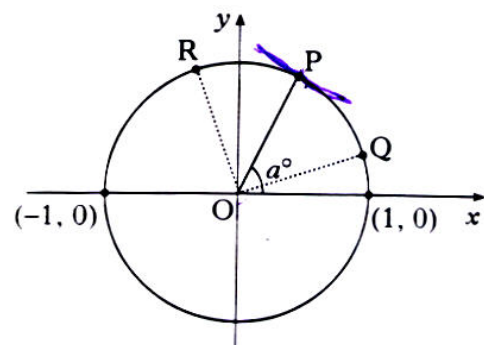
(a) Show that  $P$  is the point  $(\cos a^\circ, \sin a^\circ)$ . (1)

(b) If  $\hat{POQ} = 45^\circ$ , deduce the coordinates of  $Q$  in terms of  $a$ . (1)

(c) If  $\hat{POR} = 45^\circ$ , deduce the coordinates of  $R$  in terms of  $a$ . (1)

(d) Hence find an expression for the gradient of  $QR$  in its simplest form. (4)

(e) Show that the tangent to the circle at  $P$  is parallel to  $QR$ . (2)

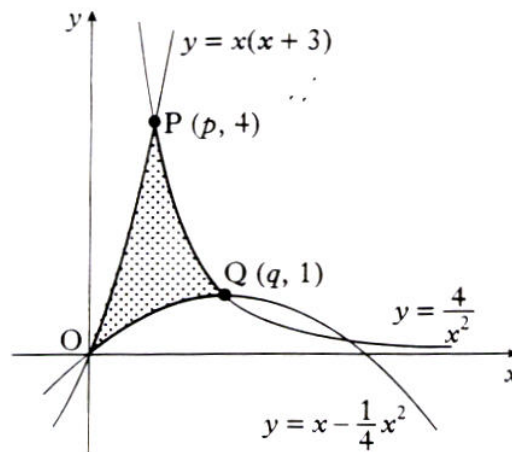


9. Solve the equation  $2\sin x^\circ - 3\cos x^\circ = 2 \cdot 5$  in the interval  $0 \leq x < 360$ . (8)

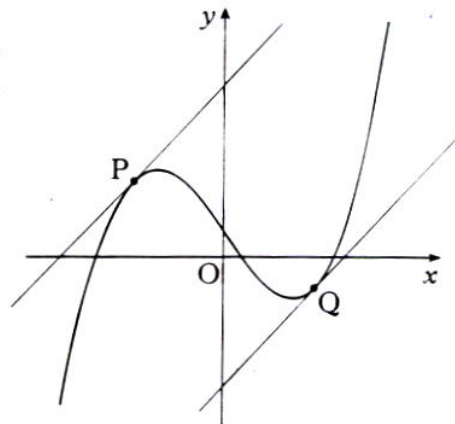
**Marks**

10. The origin, O, and the points P and Q are the vertices of a curved "triangle" which is shaded in the diagram.

The sides lie on curves with equations  $y = x(x+3)$ ,  $y = x - \frac{1}{4}x^2$  and  $y = \frac{4}{x^2}$ .



- (a) P and Q have coordinates  $(p, 4)$  and  $(q, 1)$ . Find the values of  $p$  and  $q$ . **(2)**
- (b) Calculate the shaded area. **(7)**
11. The diagram shows a sketch of the graph of  $y = x^3 - 9x + 4$  and two parallel tangents drawn at P and Q.



- (a) Find the equations of the tangents to the curve  $y = x^3 - 9x + 4$  which have gradient 3. **(6)**
- (b) Show that the shortest distance between the tangents is  $\frac{16\sqrt{10}}{5}$ . **(6)**

[END OF QUESTION PAPER]