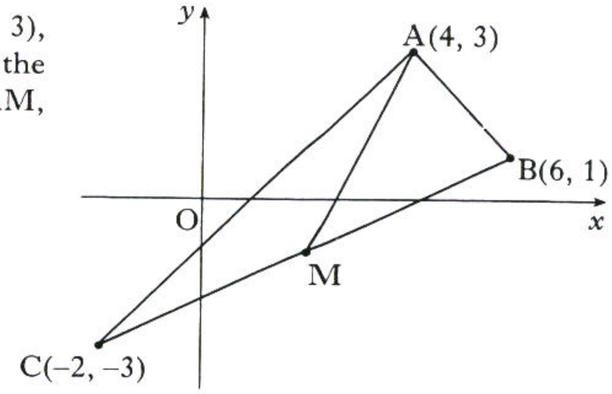
All questions should be attempted *Marks*

1. A triangle ABC has vertices A(4, 3), B(6, 1) and C(−2, −3) as shown in the diagram. Find the equation of AM, the median from A. **(3)**



2. Express $x^3 - 4x^2 - 7x + 10$ in its fully factorised form. **(4)**

3. Vectors $\boldsymbol{p}$, $\boldsymbol{q}$ and $\boldsymbol{r}$ are defined by

$$\boldsymbol{p} = \boldsymbol{i} + \boldsymbol{j} - \boldsymbol{k}, \quad \boldsymbol{q} = \boldsymbol{i} + 4\boldsymbol{k} \quad \text{and} \quad \boldsymbol{r} = 4\boldsymbol{i} - 3\boldsymbol{j}.$$

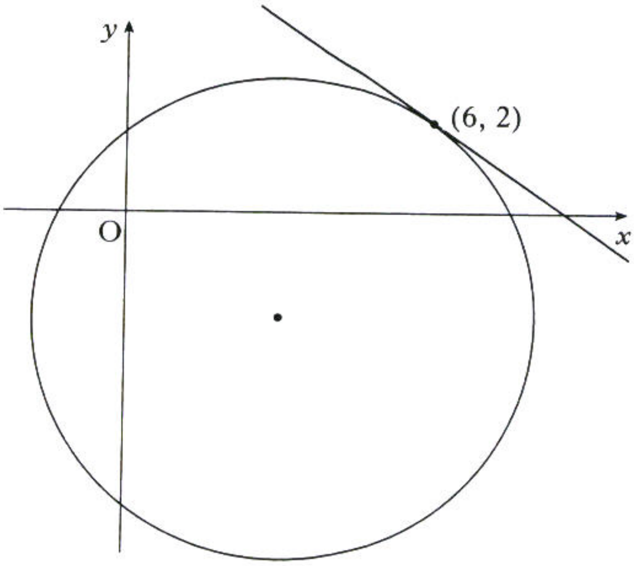
(*a*) Express $\boldsymbol{p} - \boldsymbol{q} + 2\boldsymbol{r}$ in component form. **(2)**

(*b*) Calculate $\boldsymbol{p}.\boldsymbol{r}$ **(1)**

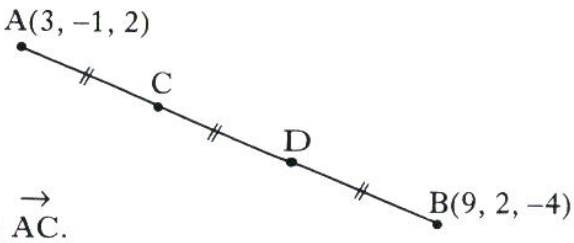
(*c*) Find $|\boldsymbol{r}|$. **(1)**

4. The circle shown has equation $(x-3)^2 + (y+2)^2 = 25$.

Find the equation of the tangent at the point (6, 2). **(4)**



5. The line AB is divided into 3 equal parts by the points C and D, as shown. A and B have coordinates (3, −1, 2) and (9, 2, −4).



(*a*) Find the components of $\overrightarrow{AB}$ and $\overrightarrow{AC}$. **(2)**

(*b*) Find the coordinates of C and D. **(2)**

Marks

6. The functions f and g are defined on a suitable domain by

$$f(x) = x^2 - 1 \text{ and } g(x) = x^2 + 2.$$

(*a*) Find an expression for $f(g(x))$. **(2)**

(*b*) Factorise $f(g(x))$. **(2)**

7. A and B are acute angles such that $\tan A = \frac{3}{4}$ and $\tan B = \frac{5}{12}$.
Find the exact value of

(*a*) $\sin 2A$ **(2)**

(*b*) $\cos 2A$ **(1)**

(*c*) $\sin(2A + B)$. **(2)**

8. Two sequences are defined by these recurrence relations

$$u_{n+1} = 3u_n - 0\cdot4 \text{ with } u_0 = 1, \qquad v_{n+1} = 0\cdot3v_n + 4 \text{ with } v_0 = 1.$$

(*a*) Explain why only one of these sequences approaches a limit as $n \to \infty$. **(1)**

(*b*) Find algebraically the exact value of the limit. **(2)**

(*c*) For the other sequence, find

(i) the smallest value of n for which the n^{th} term exceeds 1000, and

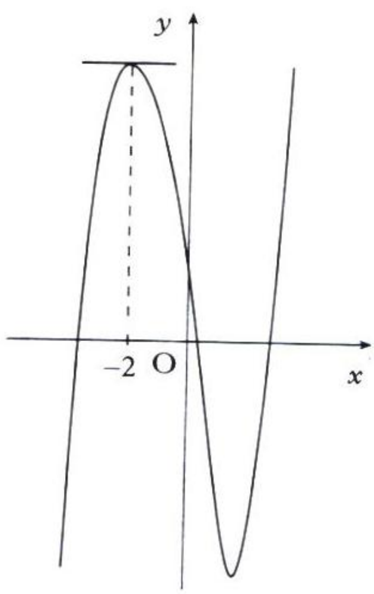
(ii) the value of that term. **(2)**

9. Solve the equation $2\sin\left(2x - \frac{\pi}{6}\right) = 1, \quad 0 \le x < 2\pi$. **(4)**

10. A curve, for which $\frac{dy}{dx} = 6x^2 - 2x$, passes through the point $(-1, 2)$.
Express y in terms of x. **(3)**

11. The diagram shows a sketch of the curve $y = x^3 + kx^2 - 8x + 3$. The tangent to the curve at $x = -2$ is parallel to the x-axis.

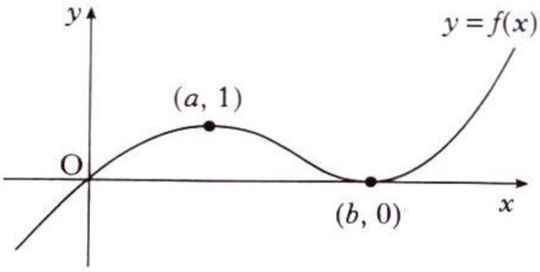
Find the value of k. **(4)**



12. Evaluate $\int_1^2 \left(x^2 + \frac{1}{x}\right)^2 dx$. **(5)**

Marks

13. A sketch of the graph of the cubic function f is shown. It passes through the origin, has a maximum turning point at $(a, 1)$ and a minimum turning point at $(b, 0)$.



(a) Make a copy of this diagram and on it sketch the graph of $y = 2 - f(x)$, indicating the coordinates of the turning points. **(3)**

(b) On a separate diagram sketch the graph of $y = f'(x)$. **(2)**

(c) The tangent to $y = f(x)$ at the origin has equation $y = \frac{1}{2}x$.

Use this information to write down the coordinates of a point on the graph of $y = f'(x)$. **(1)**

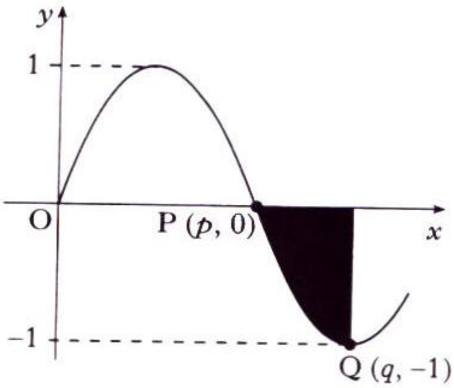
14. Differentiate $2\sqrt{x}(x+2)$ with respect to x. **(4)**

15. A sketch of part of the graph of $y = \sin 2x$ is shown in the diagram.

The points P and Q have coordinates $(p, 0)$ and $(q, -1)$.

(a) Write down the values of p and q. **(1)**

(b) Find the area of the shaded region. **(4)**



16. Given $f(x) = (\sin x + 1)^2$, find the exact value of $f'\left(\frac{\pi}{6}\right)$. **(3)**

17. A ball is thrown vertically upwards.

After t seconds its height is h metres, where $h = 1{\cdot}2 + 19{\cdot}6t - 4{\cdot}9t^2$.

(a) Find the speed of the ball after 1 second. **(3)**

(b) For how many seconds is the ball travelling upwards? **(2)**

18. (a) Write the equation $\cos 2\theta + 8\cos\theta + 9 = 0$ in terms of $\cos\theta$ and show that, for $\cos\theta$, it has equal roots. **(3)**

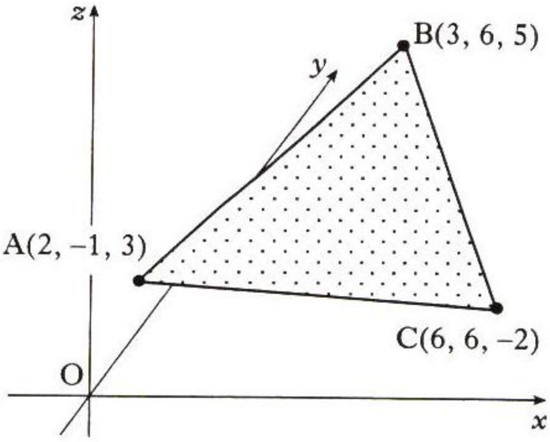
(b) Show that there are no real roots for θ. **(1)**

19. Given $x = \log_5 3 + \log_5 4$, find algebraically the value of x. **(4)**

[END OF QUESTION PAPER]

All questions should be attempted *Marks*

1. A triangle ABC has vertices A(2, –1, 3), B(3, 6, 5) and C(6, 6, –2).

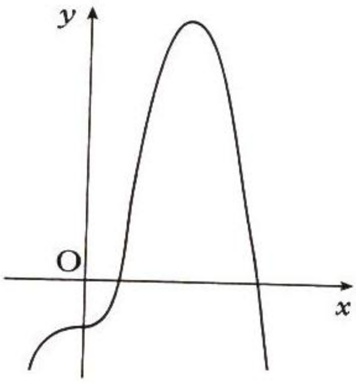


(*a*) Find $\overrightarrow{AB}$ and $\overrightarrow{AC}$. **(2)**

(*b*) Calculate the size of angle BAC. **(5)**

(*c*) Hence find the area of the triangle. **(2)**

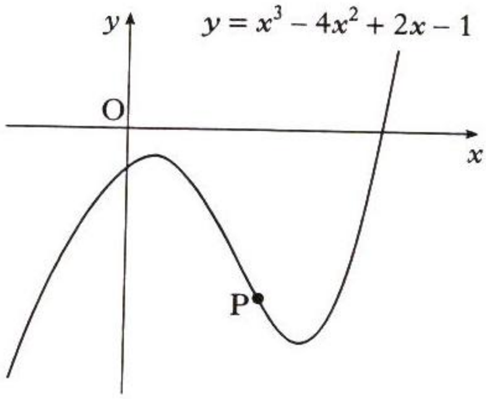
2. A curve has equation $y = -x^4 + 4x^3 - 2$. An incomplete sketch of the graph is shown in the diagram.



(*a*) Find the coordinates of the stationary points. **(6)**

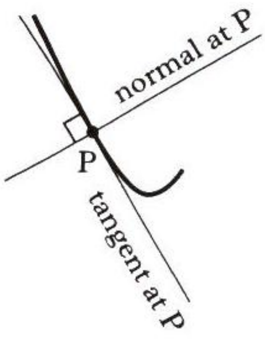
(*b*) Determine the nature of the stationary points. **(2)**

3. (*a*) The diagram shows an incomplete sketch of the curve with equation $y = x^3 - 4x^2 + 2x - 1$.



Find the equation of the tangent to the curve at the point P where $x = 2$. **(5)**

(*b*) The normal to the curve at P is defined as the straight line through P which is perpendicular to the tangent to the curve at P.

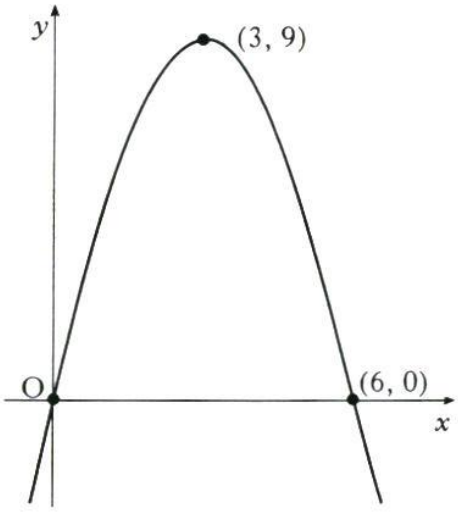


Find the angle which the normal at P makes with the positive direction of the x-axis. **(2)**

Marks

4. A parabola passes through the points (0, 0), (6, 0) and (3, 9) as shown in Diagram 1.

Diagram 1

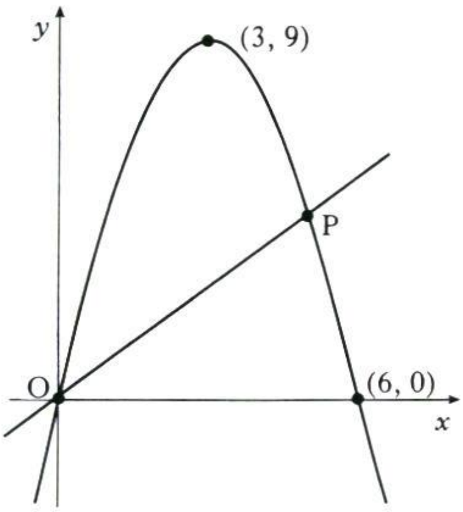


(*a*) The parabola has equation of the form $y = ax(b - x)$.

Determine the values of a and b. **(2)**

(*b*) Find the area enclosed by the parabola and the x-axis. **(4)**

Diagram 2

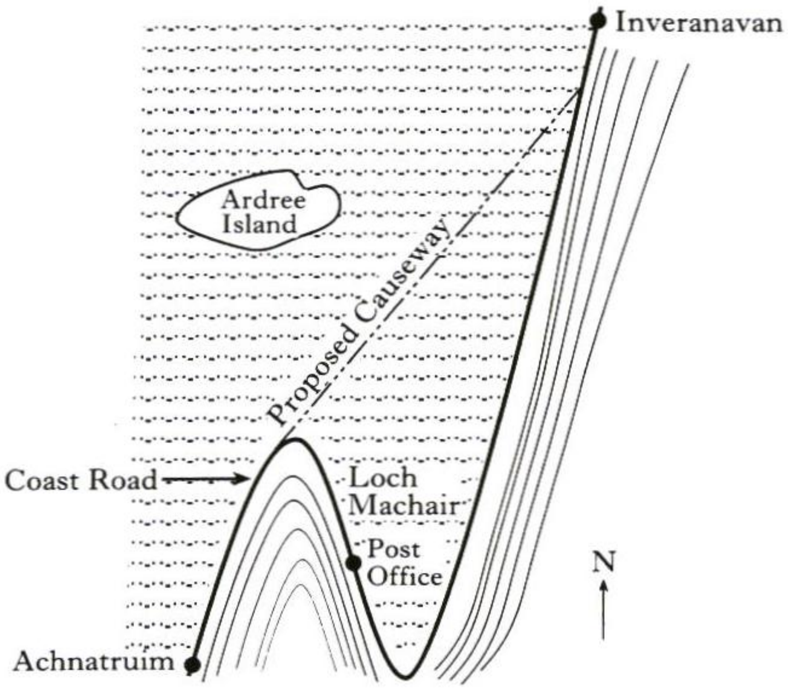


(*c*) (i) Diagram 2 shows the parabola from (*a*) and the straight line with equation $y = x$. Find the coordinates of P, the point of intersection of the parabola and the line.

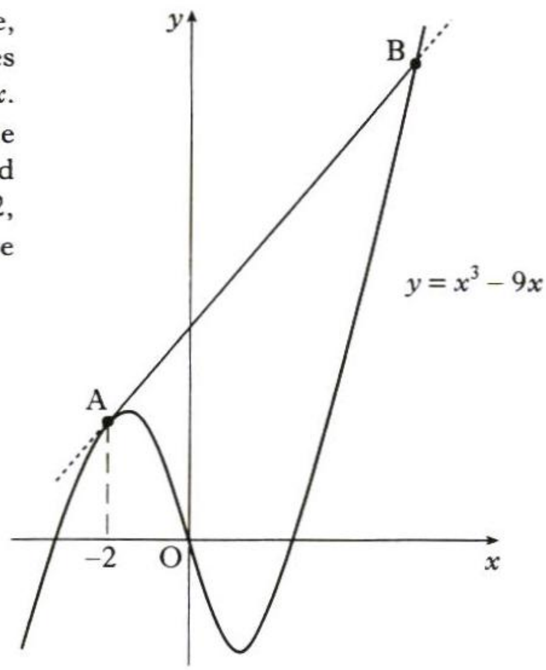
(ii) Calculate the area enclosed between the parabola and the line. **(5)**

Marks

5. The map shows part of the coast road from Achnatruim to Inveranavan. In order to avoid the hairpin bends, it is proposed to build a straight causeway, as shown, with the southern end tangential to the existing road.



With the origin taken at the Post Office, the part of the coast road shown lies along the curve with equation $y = x^3 - 9x$. The causeway is represented by the line AB. The southern end of the proposed causeway is at the point A where $x = -2$, and the line AB is a tangent to the curve at A.



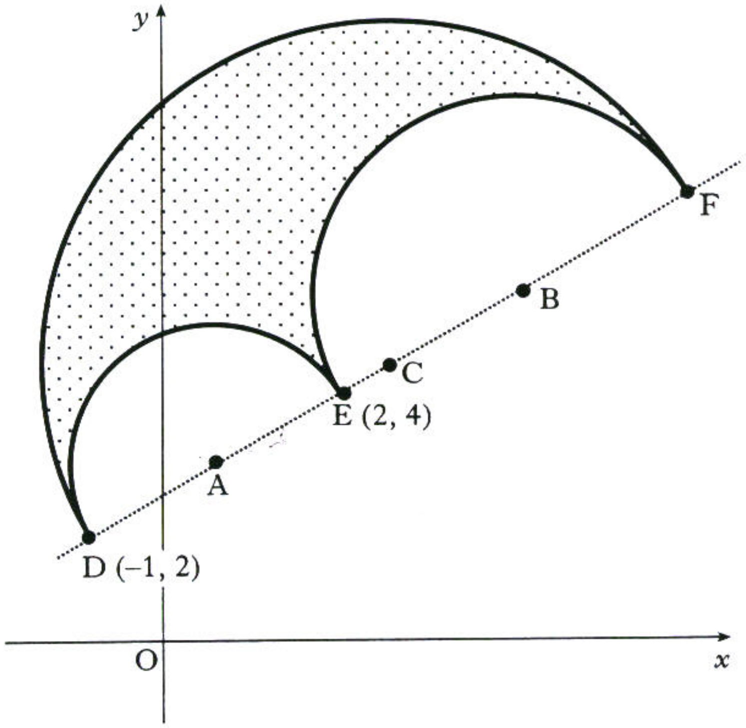
(*a*) (i) Write down the coordinates of A.

(ii) Find the equation of the line AB. **(5)**

(*b*) Determine the coordinates of the point B which represents the northern end of the causeway. **(7)**

Marks

6. The perimeter of the shape shown in the diagram is composed of 3 semicircles with centres A, B and C which lie on a straight line.



DE is a diameter of one of the semicircles. The coordinates of D and E are (−1, 2) and (2, 4).

(*a*) Find the equation of the circle with centre A and diameter DE. **(3)**

The circle with centre B and diameter EF has equation

$$x^2 + y^2 - 16x - 16y + 76 = 0.$$

(*b*) (i) Write down the coordinates of B.

(ii) Determine the coordinates of F and C. **(3)**

(*c*) In the diagram the perimeter of the shape is represented by the thick black line.

Show that the perimeter is $5\pi\sqrt{13}$ units. **(3)**

Marks

7. The function f is defined by $f(x) = 2\cos x° - 3\sin x°$.

(*a*) Show that $f(x)$ can be expressed in the form $f(x) = k\cos(x + \alpha)°$ where $k > 0$ and $0 \leq \alpha < 360$, and determine the values of k and α. **(4)**

(*b*) Hence find the maximum and minimum values of $f(x)$ and the values of x at which they occur, where x lies in the interval $0 \leq x < 360$. **(4)**

(*c*) Write down the minimum value of $(f(x))^2$. **(1)**

8. A gardener feeds her trees weekly with "Bioforce, the wonder plant food". It is known that in a week the amount of plant food in the tree falls by about 25%.

(*a*) The trees contain no Bioforce initially and the gardener applies 1 g of Bioforce to each tree every Saturday. Bioforce is only effective when there is continuously more than 2 g of it in the tree. Calculate how many weekly feeds will be necessary before the Bioforce becomes effective. **(3)**

(*b*) (i) Write down a recurrence relation for the amount of plant food in the tree immediately after feeding. **(1)**

(ii) If the level of Bioforce in the tree exceeds 5 g, it will cause leaf burn. Is it safe to continue feeding the trees at this rate indefinitely? **(4)**

[Turn over

Marks

9. Diagram 1 shows the area between the line $y = 3$ and the x-axis from $x = a$ to $x = b$. If this area is rotated through 360° about the x-axis, it forms a solid shape (a cylinder) as shown in Diagram 2.

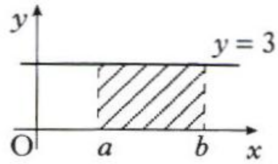


Diagram 1

The volume of this solid may be obtained by evaluating the integral $\pi \int_a^b y^2 dx$.

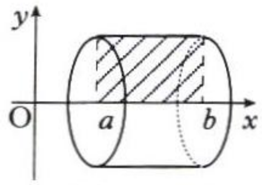


Diagram 2

Worked Example

The area between $y = 2x$ and the x-axis from $x = 1$ to $x = 3$ is rotated about the x-axis. The volume of the solid is calculated as follows:

$$y = 2x$$

$$y^2 = (2x)^2 = 4x^2$$

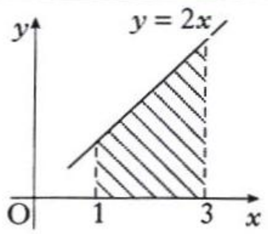
$$\pi \int_1^3 y^2 dx$$

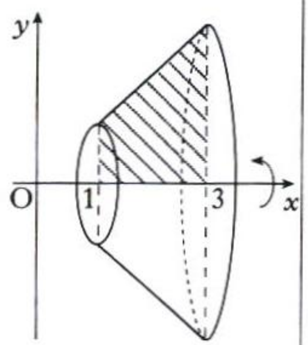
$$= \pi \int_1^3 4x^2 dx$$

$$= \pi \left[\frac{4}{3}x^3\right]_1^3$$

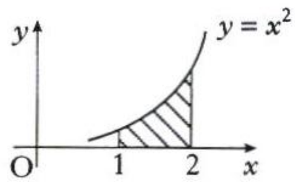
$$= \pi \left[36 - \frac{4}{3}\right]$$

$$\text{Volume} = \frac{104}{3}\pi \text{ units}^3$$

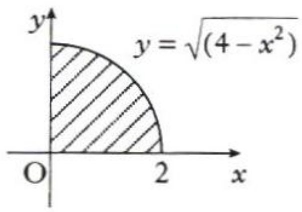




(*a*) Use this method to find the volume of the solid formed when the area between $y = x^2$ and the x-axis from $x = 1$ to $x = 2$ is rotated about the x-axis. **(4)**



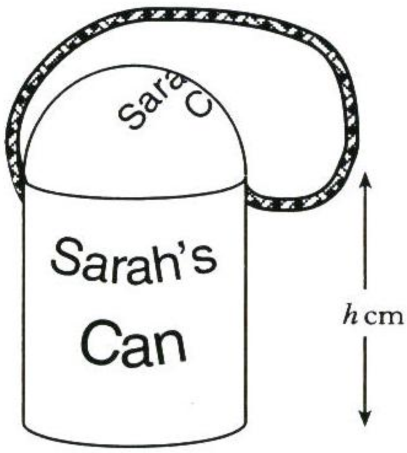
(*b*) (i) Use this method to find the volume of the solid formed when the area between $y = \sqrt{(4 - x^2)}$ and the x-axis from $x = 0$ to $x = 2$ is rotated about the x-axis. **(4)**



(ii) **Hence** write down the volume of a sphere of radius 2. **(1)**

Marks

10. A child's drinking beaker is in the shape of a cylinder with a hemispherical lid and a circular flat base. The radius of the cylinder is r cm and the height is h cm. The volume of the cylinder is $400\ \text{cm}^3$.



(*a*) Show that the surface area of plastic, $A(r)$, needed to make the beaker is given by $A(r) = 3\pi r^2 + \dfrac{800}{r}$. **(3)**

Note: The curved surface area of a hemisphere of radius r is $2\pi r^2$.

(*b*) Find the value of r which ensures that the surface area of plastic is minimised. **(6)**

11. (*a*) The variables x and y are connected by a relationship of the form $y = ae^{bx}$ where a and b are constants. Show that there is a linear relationship between $\log_e y$ and x. **(3)**

(*b*) From an experiment some data was obtained. The table shows the data which lies on the line of best fit.

x	3·1	3·5	4·1	5·2
y	21 876	72 631	439 392	11 913 076

The variables x and y in the above table are connected by a relationship of the form $y = ae^{bx}$. Determine the values of a and b. **(6)**

[*END OF QUESTION PAPER*]

[2500/202]