S.C.E. HIGHER ANSWERS – 1996

PAPER I

1. $3x + 2y = 17$

2. $a = 2\frac{1}{2}$

3. $\left(\frac{5\pi}{6}, 3\right)$

4. $2x + y = 10$

5. 9

6. Verify that $\vec{AB} = \begin{pmatrix}4\\2\\-2\end{pmatrix}$, $\vec{BC} = \begin{pmatrix}6\\3\\-3\end{pmatrix}$

   $\therefore$ $\vec{AB} = \frac{2}{3}\vec{BC}$

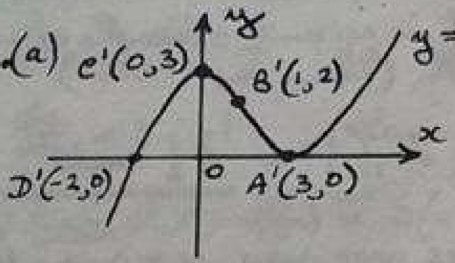
   $\therefore$ AB is parallel to BC

   $\therefore$ A, B, C are collinear [$\because$ B is a common point]

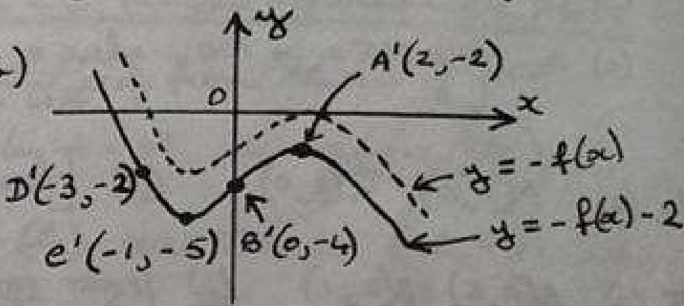
   B divides AC in the ratio $2:3$ (i.e. $AB : BC = 2 : 3$)

7. $x(x-1)(x^2+x+1)$

8. (a)



(b)



9. $f'(4) = \frac{5}{16}$

10. $0°, 120°, 180°, 240°$ [NB NOT $360°$ since $0° \leq x < 360°$]

11. (a) Limit exists because 0.3 lies between -1 and 1 (b) $7\frac{1}{7}$

12. $P\left(\frac{\pi}{6}, \frac{1}{2}\right)$ $Q\left(\frac{\pi}{2}, \frac{1}{2}\right)$

13. $\frac{dy}{dx} = \frac{-\sin x}{2\sqrt{1+\cos x}}$

14. Take 2 lines $\left\{\begin{matrix}2x + 3y = 4\\3x - y = 17\end{matrix}\right\}$. Verify that they intersect at $(5, -2)$.
    Then verify that $(5, -2)$ does NOT lie on the 3rd line $x - 3y - 10 = 0$
    $\therefore$ 3 LINES ARE NOT CONCURRENT.

15. [Verify that $BD = 5$ and $AD = 2\sqrt{6}$; then use the expansion of $\cos(x+y)$]

16. f is increasing when $x < -2$ AND $x > 3$ [TABLE OF VALUES REQUIRED]

17. $4(x+1)^2 - 9$

18. $\sin x = \frac{4\sqrt{5}}{9}$

19. (a) $k = 0.019$ (b) [FALLS TO $56.56°C$ IN NEXT 15 MINS]
    $\therefore$ IT FALLS $18.44°C$ IN NEXT 15 MINS

20. $(x-3)^2 + (y-4)^2 = 25$.

PAPER II

1. (a) S.P.'s are $(0, 3)$ AND $(3, -24)$ (b) $(0, 3)$ is a point of inflection
   $(3, -24)$ is a MINIMUM S.P.

2. (a) Verify that $m_{AB} m_{BC} = -1$ WITH A CLEAR CONCLUSION $\left(\begin{matrix}m_{AB} = 2\\m_{BC} = -\frac{1}{2}\end{matrix}\right)$

   (b)(i) AD: $x - 3y = 6$
   BE: $4x + 3y = -1$

   (ii) $M\left(1, -\frac{5}{3}\right)$

(a) $Q(2,2,9)$ $\quad R(21,3,12)$ $\qquad$ (b) $Q\hat{P}R = 83 \cdot 4^\circ$

(a) (i) $g[f(x)] = 4x^2 + 4x + 1 + k$ $\quad$ (ii) $f[g(x)] = 2x^2 + 2k + 1$

(b) (i) [ANSWER IN QUESTION] $\quad$ (ii) $[\Delta = 64]$ $\therefore$ Roots are REAL AND DISTINCT

(iii) $k = -2$

5. $\left[\text{area I} = \frac{1}{2} - \frac{\sqrt{3}}{4} \ ; \ \text{area II} = \frac{1}{2}\right]$ $\therefore$ TOTAL AREA $= 1 - \frac{\sqrt{3}}{4}$ units^2

6. (a) Line l : $y = 2x$ $\quad$ Parabola p : $y = 2x^2$ $\quad$ Circle c : $x^2 + y^2 = 5$

(b) l' : $y = 2x - 4$ $\quad$ p' : $y = -2x^2$ $\quad$ c' : $(x-2)^2 + y^2 = 5$

(c) $(-2,-8)$ AND $(1,-2)$

7. (a) $\sqrt{13}\cos(x - 56 \cdot 3^\circ)$ $\qquad$ (b) $138 \cdot 3^\circ$, $334 \cdot 3^\circ$

8. (a) $5x + y = -3$ $\ ;\ $ $B(-1,2)$

(b) $\frac{4}{3}$ units^2

9. (a) $Y = 3X + 0 \cdot 7$

(b) [FIRST REPLACE Y BY $\log_e y$ AND X BY $\log_e x$ TO OBTAIN :- $y = 2 \cdot 014 x^3$.] $\therefore \begin{Bmatrix} a = 2 \cdot 014 \\ b = 3 \end{Bmatrix}$

10. (a) By symmetry, result needs only to be proved for one point at $(3,7)$: PROVE THAT TANGENT TO PARABOLA HAS GRADIENT $\frac{2}{3}$ AND PROVE THAT TANGENT TO CIRCLE HAS GRADIENT $-3/2$. Then conclude that since product of gradients of tangents is -1, the tangents are perpendicular. $\therefore$ CURVES ARE ORTHOGONAL.

(b) (i) $2y = 3x + 23$

(ii) $x^2 + (y - 11\frac{1}{2})^2 = 29\frac{1}{4}$

11. (a) (i) $h = 5 - x - \frac{1}{2}\pi x$

(ii) [START WITH $L = 2(\text{area of rectangle}) + 1(\text{area of } \tfrac{1}{2} \text{ circle})$]

(b) $x = \dfrac{20}{8 + 3\pi}$ $(\doteq 1 \cdot 1478)$

$h = \dfrac{20 + 5\pi}{8 + 3\pi}$ $(\doteq 2 \cdot 0493)$

NB : TABLE OF VALUES IS REQUIRED