National
Qualifications
2026

2026 Mathematics

Advanced Higher Paper 2

Question Paper Finalised Marking Instructions

Qualifications Scotland
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General marking principles for Advanced Higher Mathematics

Always apply these general principles. Use them in conjunction with the detailed marking instructions, which identify the key features required in candidates' responses.

For each question, the marking instructions are generally in two sections:

- *generic scheme — this indicates why each mark is awarded*
- *illustrative scheme — this covers methods which are commonly seen throughout the marking*

In general, you should use the illustrative scheme. Only use the generic scheme where a candidate has used a method not covered in the illustrative scheme.

(a) Always use positive marking. This means candidates accumulate marks for the demonstration of relevant skills, knowledge and understanding; marks are not deducted for errors or omissions.

(b) If you are uncertain how to assess a specific candidate response because it is not covered by the general marking principles or the detailed marking instructions, you must seek guidance from your team leader.

(c) One mark is available for each •. There are no half marks.

(d) If a candidate's response contains an error, all working subsequent to this error must still be marked. Only award marks if the level of difficulty in their working is similar to the level of difficulty in the illustrative scheme.

(e) Only award full marks where the solution contains appropriate working. A correct answer with no working receives no mark, unless specifically mentioned in the marking instructions.

(f) Candidates may use any mathematically correct method to answer questions, except in cases where a particular method is specified or excluded.

(g) If an error is trivial, casual or insignificant, for example $6 \square 6 = 12$, candidates lose the opportunity to gain a mark, except for instances such as the second example in point (h) below.

(h) If a candidate makes a transcription error (question paper to script or within script), they lose the opportunity to gain the next process mark, for example

This is a transcription error and so the mark is not awarded.

This is no longer a solution of a quadratic equation, so the mark is not awarded.

$$x^2 + 5x + 7 = 9x + 4$$
$$x - 4x + 3 = 0$$
$$x = 1$$

The following example is an exception to the above

This error is not treated as a transcription error, as the candidate deals with the intended quadratic equation. The candidate has been given the benefit of the doubt and all marks awarded.

$$x^2 + 5x + 7 = 9x + 4$$
$$x - 4x + 3 = 0$$
$$(x-3)(x-1) = 0$$
$$x = 1 \text{ or } 3$$

(i) **Horizontal/vertical marking**
If a question results in two pairs of solutions, apply the following technique, but only if indicated in the detailed marking instructions for the question.

Example:

	$\bullet^5$	$\bullet^6$
$\bullet^5$	$x=2$	$x=-4$
$\bullet^6$	$y=5$	$y=-7$

Horizontal: $\bullet^5$ $x=2$ and $x=-4$ Vertical: $\bullet^5$ $x=2$ and $y=5$
$\bullet^6$ $y=5$ and $y=-7$ $\bullet^6$ $x=-4$ and $y=-7$

You must choose whichever method benefits the candidate, **not** a combination of both.

(j) In final answers, candidates should simplify numerical values as far as possible unless specifically mentioned in the detailed marking instruction. For example

$\frac{15}{12}$ must be simplified to $\frac{5}{4}$. or $1\frac{1}{4}$ $\frac{43}{1}$ must be simplified to 43

$\frac{15}{0.3}$ must be simplified to 50 $\frac{4/5}{3}$ must be simplified to $\frac{4}{15}$

$\sqrt{64}$ must be simplified to 8*

*The square root of perfect squares up to and including 144 must be known.

(k) Commonly Observed Responses (COR) are shown in the marking instructions to help mark common and/or non-routine solutions. CORs may also be used as a guide when marking similar non-routine candidate responses.

(l) Do not penalise candidates for any of the following, unless specifically mentioned in the detailed marking instructions:

- working subsequent to a correct answer
- correct working in the wrong part of a question
- legitimate variations in numerical answers/algebraic expressions, for example angles in degrees rounded to nearest degree
- omission of units
- bad form (bad form only becomes bad form if subsequent working is correct), for example
  $(x^3+2x^2+3x+2)(2x+1)$ written as
  $(x^3+2x^2+3x+2)\times 2x+1$
  $=2x^4+5x^3+8x^2+7x+2$ gains full credit
- repeated error within a question, but not between questions or papers

(m) In any 'Show that…' question, where candidates have to arrive at a required result, the last mark is not awarded as a follow-through from a previous error, unless specified in the detailed marking instructions.

(n) You must check all working carefully, even where a fundamental misunderstanding is apparent early in a candidate's response. You may still be able to award marks later in the question so you must refer continually to the marking instructions. The appearance of the correct answer does not necessarily indicate that you can award all the available marks to a candidate.

(o) You should mark legible scored-out working that has not been replaced. However, if the scored-out working has been replaced, you must only mark the replacement working.

(p) If candidates make multiple attempts using the same strategy and do not identify their final answer, mark all attempts and award the lowest mark. If candidates try different valid strategies, apply the above rule to attempts within each strategy and then award the highest mark.

For example:

Strategy 1 attempt 1 is worth 3 marks.	Strategy 2 attempt 1 is worth 1 mark.
Strategy 1 attempt 2 is worth 4 marks.	Strategy 2 attempt 2 is worth 5 marks.
From the attempts using strategy 1, the resultant mark would be 3.	From the attempts using strategy 2, the resultant mark would be 1.

In this case, award 3 marks.

Marking Instructions for each question

Question			Generic scheme	Illustrative scheme	Max mark
1.			•[1] start differentiation •[2] apply chain rule	•[1] $\dfrac{3}{\sqrt{1-(7x)^2}}$ •[2] $\dfrac{21}{\sqrt{1-(7x)^2}}$	2
Notes:					
Commonly Observed Responses:					

Question			Generic scheme	Illustrative scheme	Max mark
2.			•¹ write binomial expansion [1,2,3,4,5]	•¹ $\binom{4}{0}(x^2)^4+\binom{4}{1}(x^2)^3\left(-\frac{5}{x}\right)$ $+\binom{4}{2}(x^2)^2\left(-\frac{5}{x}\right)^2+\binom{4}{3}(x^2)\left(-\frac{5}{x}\right)^3$ $+\binom{4}{4}\left(-\frac{5}{x}\right)^4$	**4**
			•² resolve signs [1,2,3,4,5]	•²˒³˒⁴ $x^8-20x^5+150x^2-\frac{500}{x}+\frac{625}{x^4}$	
			•³ simplify coefficients or powers of x [1,2,3,4,5,6]		
			•⁴ complete simplification and obtain expression [1,2,3,4,5,6]		

Notes:

1. Accept any correct form for binomial coefficients; disregard poor notation where the correct numerical values subsequently appear. If binomial coefficients are missing, •²,•³ and •⁴ are still available.
2. Where a candidate writes $\sum_{r=0}^{4}\binom{4}{r}(x^2)^{4-r}\left(\frac{-5}{x}\right)^r$, award •¹ only, unless they continue to produce a full expansion. In cases where the sigma notation precedes a full expansion, disregard incorrect or absent limits.
3. Candidates who expand $\left(x^2-\frac{5}{x}\right)^4$ without using the binomial theorem may be awarded •²,•³ and •⁴ but •¹ is not available. Evidence of use of the binomial theorem may include a systematic identification of coefficients and powers of individual terms, eg expressed in a table.
4. For candidates who expand $\left(x^2+\frac{5}{x}\right)^4$ using the binomial theorem •¹ and •² are not available.
5. Where a candidate omits one term, or evaluates one term to zero, the maximum mark is 2/4. Where this occurs at •¹, withhold •¹ and •⁴; where it occurs later, •³ and •⁴ are not available. If two or more terms are omitted or equated to zero, only •¹ is available.
6. Accept negative powers of x and eg $500\cdot\frac{1}{x}$.

Commonly Observed Responses:

$x^8-20x^5+150x^2-\frac{500}{x}+\frac{625}{4}$

Award ·¹ and ·² only.

Question			Generic scheme	Illustrative scheme	Max mark
3.	(a)	(i)	**Method 1** •¹ all three derivatives AND all four evaluations [1,2] •² obtain simplified expression [2,3]	**Method 1** $f(x)=e^{3x}$ $f(0)=1$ •¹ $f'(x)=3e^{3x}$ $f'(0)=3$ $f''(x)=9e^{3x}$ $f''(0)=9$ $f'''(x)=27e^{3x}$ $f'''(0)=27$ •² $1+3x+\frac{9}{2}x^2+\frac{9}{2}x^3$	2
			Method 2 •¹ write down Maclaurin series for e^x [1,2] •² substitute and simplify [2,3]	**Method 2** •¹ $1+x+\frac{x^2}{2!}+\frac{x^3}{3!}$ stated or implied •² $1+3x+\frac{9}{2}x^2+\frac{9}{2}x^3$	

Notes:

1. Where notation is absent or incomplete, evidence for •¹ may appear as substitution of values into the Maclaurin expansion but see COR below.
2. Where a candidate produces a correct expansion without working, award 2/2.
3. The simplification for •² may appear in (b).

Commonly Observed Responses:

Leaving variable in expansion:

$$e^{3x}+3xe^{3x}+\frac{9x^2e^{3x}}{2}+\frac{27x^3e^{3x}}{6}$$

$$=e^{3(0)}+3xe^{3(0)}+\frac{9x^2e^{3(0)}}{2}+\frac{27x^3e^{3(0)}}{6}$$

$$=1+3x+\frac{9}{2}x^2+\frac{9}{2}x^3$$

Do not award •¹.

Question			Generic scheme	Illustrative scheme	Max mark
3.	(a)	(ii)	•³ all three derivatives AND all four evaluations [1,2] •⁴ obtain simplified expression [2,3]	$f(x)=\ln(1+x)$ $\quad f(0)=0$ $f'(x)=\frac{1}{1+x}$ $\quad f'(0)=1$ •³ $f''(x)=-\frac{1}{(1+x)^2}$ $\quad f''(0)=-1$ $f'''(x)=\frac{2}{(1+x)^3}$ $\quad f'''(0)=2$ •⁴ $x-\frac{x^2}{2}+\frac{x^3}{3}$	**2**

Notes:

1. Where notation is absent or incomplete, evidence for •³ may appear as substitution of values into the Maclaurin expansion.
2. Where a candidate produces a correct expansion without working, award 2/2.
3. The simplification for •⁴ may appear in (b), either in the original expression or in the expression for $\ln\left(\frac{1}{1+x}\right)$.

Commonly Observed Responses:

	(b)		•⁵ determine series for $\ln\left(\frac{1}{1+x}\right)$ and express as a product [1] •⁶ expand and simplify [2,3]	•⁵ $\left(1+3x+\frac{9}{2}x^2+\frac{9}{2}x^3\right)\left(-x+\frac{x^2}{2}-\frac{x^3}{3}\right)$ •⁶ $-x-\frac{5}{2}x^2-\frac{10}{3}x^3$	**2**

Notes:

1. Award •⁵ for $-\left(1+3x+\frac{9}{2}x^2+\frac{9}{2}x^3\right)\left(x-\frac{x^2}{2}+\frac{x^3}{3}\right)$ or $\left(1+3x+\frac{9}{2}x^2+\frac{9}{2}x^3\right)\left(\ln 1-\left(x-\frac{x^2}{2}+\frac{x^3}{3}\right)\right)$
2. For the award of •⁶:
   a. There must be at least three (including a constant and a cubed) terms in the exponential expansion and at least two (including a cubed) terms in the log expansion.
   b. There must be no negative powers of x.
3. Disregard higher powers of x.

Question		Generic scheme	Illustrative scheme	Max mark
3.	(b)	(continued)		

Commonly Observed Responses:

COR A - From $e^{3x}\ln\left(\dfrac{1}{1+x}\right)$

$$f'(x)=3e^{3x}\ln\left(\frac{1}{1+x}\right)-e^{3x}\left(1+x\right)^{-1}$$
$$f''(x)=9e^{3x}\ln\left(\frac{1}{1+x}\right)-6e^{3x}\left(1+x\right)^{-1}+e^{3x}(1+x)^{-2}$$
$$f'''(x)=27e^{3x}\ln\left(\frac{1}{1+x}\right)-27e^{3x}(1+x)^{-1}+9e^{3x}(1+x)^{-2}-2e^{3x}(1+x)^{-3}$$

$f(0)=0$
$f'(0)=-1$
$f''(0)=-5$
$f'''(0)=-20$

Award $\bullet^5$ for the first two derivatives and first three evaluations.

COR B - From $-e^{3x}\ln\left(1+x\right)$

$$f'(x)=-3e^{3x}\ln\left(1+x\right)-e^{3x}\left(1+x\right)^{-1}$$
$$f''(x)=-9e^{3x}\ln\left(1+x\right)-6e^{3x}\left(1+x\right)^{-1}+e^{3x}(1+x)^{-2}$$
$$f'''(x)=-27e^{3x}\ln\left(1+x\right)-27e^{3x}(1+x)^{-1}+9e^{3x}(1+x)^{-2}-2e^{3x}(1+x)^{-3}$$

Award marks as for COR A.

Question			Generic scheme	Illustrative scheme	Max mark
4.	(a)		•[1] obtain and identify d [1,2]	•[1] eg $1428 = 2\times 567 + 294$, leading to eg $\underline{21}$ **or** gcd$=21$	1

Notes:

1. The gcd must be clearly identified in (a) or, if not identified in (a), implied by its use in (b).
2. There must be evidence of the Euclidean algorithm or a related process.

Commonly Observed Responses:

	(b)		•[2] express gcd in terms of 567 and 294 [1,2,3] •[3] determine a and b [1,3,4,5]	•[2] $21 = 294 - 1\times(567 - 1\times 294)\ldots$ •[3] $a = 2$, $b = -5$	2

Notes:

1. For the correct answer with no working, award 0/2.
2. At •[2], the expression may involve 567, 294 and 1428.
3. Where a candidate misidentifies the gcd in part (a), •[2] and •[3] are still available for the appearance of "21= ..." accompanied by appropriate working. Otherwise, award 0/2.
4. At •[3], the minimum requirement is $1428\times(2) + 567\times(-5) = 21$.
5. Do not accept $21 = 2\times 1428 - 5\times 567$ as a final answer.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
5.			•¹ take logarithms on both sides and apply rule [1,2] •² differentiate $\ln y$ [2,3,4] •³ evidence of product rule with one term correct [2,3,4,5,6] •⁴ complete differentiation [2,3,4,5,6] •⁵ write $\frac{dy}{dx}$ in terms of x [1,2,3,4,6]	•¹ $\ln y = 4x\ln x$ •² $\frac{1}{y}\frac{dy}{dx}$ •³ $4\ln x + \ldots$ **or** $\ldots + 4x\cdot\frac{1}{x}$ •⁴ $4\ln x + 4x\cdot\frac{1}{x}$ •⁵ $\frac{dy}{dx} = x^{4x}\left(4\ln x + 4\right)$	5

Notes:

1. Accept 'log' as an alternative to '$\ln$' provided a candidate does not indicate a base other than e. For candidates who do use a base other than e, only •¹ and •⁵ are available.
2. For candidates who write $y = e^{4x}\ln x$, do not award •¹,•² or •⁵. However, •³ and •⁴ are still available for a correct application of the product rule.
3. Where a candidate uses $y = xe^{4x}$, do not award •² or •⁵. However, •³ and •⁴ are still available for a correct application of the product rule.
4. Where a candidate expresses the intention to integrate:
   a. If they then differentiate both sides, •², •³, •⁴ and •⁵ are still available.
   b. If they integrate both sides, •², •³, •⁴ and •⁵ are not available.
   c. If they integrate the RHS but differentiate the LHS, •², •³ and •⁴ are not available.
5. For candidates who do not attempt to use the product rule, •³ and •⁴ are not available.
6. Accept '4' instead of '$4x\cdot\frac{1}{x}$' at •³ and •⁴. However, do not accept '$4x\cdot\frac{1}{x}$' at •⁵.

Commonly Observed Responses:

Candidates who write $y = e^{\ln x^{4x}}$:

•¹ write in the form $y = e^{\ln x^{4x}}$.

•²,³ apply chain rule $\frac{dy}{dx} = e^{4x\ln x}\cdot\frac{d}{dx}\left(4x\ln x\right)$

•⁴ use product rule with one term correct $4\ln x + \ldots$ or $\ldots + 4x\cdot\frac{1}{x}$

•⁵ complete differentiation $\frac{dy}{dx} = x^{4x}\left(4\ln x + 4\right)$ or $\frac{dy}{dx} = e^{4x\ln x}\left(4\ln x + 4\right)$

<table>
<tr><th colspan="3">Question</th><th>Generic scheme</th><th>Illustrative scheme</th><th>Max mark</th></tr>
<tr><td>6.</td><td>(a)</td><td>(i)</td><td>$\bullet^1$ obtain value of d [1,2]</td><td>$\bullet^1$ $\frac{1}{2}$</td><td>1</td></tr>
<tr><td colspan="6">Notes:
1. Do not award $\bullet^1$ if a correct value is produced from incorrect working.
2. Do not withhold $\bullet^1$ for the appearance of “$11d - 3d = \ldots$” unless eg “$a + 11d = 10$” appears.</td></tr>
<tr><td colspan="6">Commonly Observed Responses:</td></tr>
<tr><td></td><td></td><td>(ii)</td><td>$\bullet^2$ obtain value of a [1]</td><td>$\bullet^2$ 5</td><td>1</td></tr>
<tr><td colspan="6">Notes:
1. Do not award $\bullet^2$ if a correct value is produced from incorrect working.</td></tr>
<tr><td colspan="6">Commonly Observed Responses:</td></tr>
<tr><td></td><td></td><td>(iii)</td><td>$\bullet^3$ obtain value of S_{109}</td><td>$\bullet^3$ 3488</td><td>1</td></tr>
<tr><td colspan="6">Notes:</td></tr>
<tr><td colspan="6">Commonly Observed Responses:</td></tr>
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<tr><th colspan="3">Question</th><th>Generic scheme</th><th>Illustrative scheme</th><th>Max mark</th></tr>
<tr><td>6.</td><td>(b)</td><td>(i)</td><td>•⁴ obtain value of r [1,2]</td><td>•⁴ $\frac{3}{2}$</td><td>1</td></tr>
<tr><td colspan="6">Notes:
1. Do not award •⁴ if a correct value is produced from incorrect working.
2. Do not withhold •⁴ for the appearance of “$\frac{ar^4}{ar^3}=\ldots$” unless eg “$ar^3=18$” appears.</td></tr>
<tr><td colspan="6">Commonly Observed Responses:</td></tr>
<tr><td></td><td></td><td>(ii)</td><td>•⁵ obtain value of a [1]</td><td>•⁵ 8</td><td>1</td></tr>
<tr><td colspan="6">Notes:
1. Do not award •⁵ if a correct value is produced from incorrect working.</td></tr>
<tr><td colspan="6">Commonly Observed Responses:</td></tr>
<tr><td></td><td></td><td>(iii)</td><td>•⁶ obtain expression for S_n [1,2]</td><td>•⁶ $16\left(\left(\frac{3}{2}\right)^n-1\right)$</td><td>1</td></tr>
<tr><td colspan="6">Notes:
1. At •⁶ accept $-16\left(1-\left(\frac{3}{2}\right)^n\right)$
2. At •⁶ accept $16\left(\frac{3^n}{2^n}-1\right)$ but do not accept $16\left(\frac{3^n}{2}-1\right)$, unless the correct algebraic expression appears later.</td></tr>
<tr><td colspan="6">Commonly Observed Responses:</td></tr>
<tr><td></td><td>(c)</td><td></td><td>•⁷ obtain value of n [1,2,3]</td><td>•⁷ 14</td><td>1</td></tr>
<tr><td colspan="6">Notes:
1. For the award of •⁷, there must be evidence of logarithms to calculate n.
2. Where a candidate produces a value of r associated with a sum to infinity which is less than or equal to their answer to (a)(iii), •⁷ is not available.
3. Do not accept $n\geq 14$.</td></tr>
<tr><td colspan="6">Commonly Observed Responses:</td></tr>
</table>

Question			Generic scheme	Illustrative scheme	Max mark
7.	(a)		•¹ find $\frac{dx}{dt}$ •² find and simplify $\frac{dy}{dx}$	•¹ $\frac{dx}{dt}=\frac{6}{2t+1}$ stated/implied by •² •² $\frac{dy}{dx}=\frac{(1-t)(2t+1)}{6}$	**2**

Notes:

Commonly Observed Responses:

	(b)		•³ obtain one value of t •⁴ find second solution, discard negative value for t and state coordinates	•³ $t=1$ **or** $t=-\frac{1}{2}$ •⁴ $t=1$, $\cancel{t=-\frac{1}{2}}$, $\left(3\ln 3, \frac{1}{2}\right)$	**2**

Notes:

1. For •⁴, accept $x = 3\ln 3,\ y=\frac{1}{2}$.
2. Accept decimal equivalent to at least 2 significant figures (3.3).
3. Where a candidate produces two or more points, •⁴ is not available.
4. For •⁴, the rejection of $t=-\frac{1}{2}$ may be communicated by eg $t>0$. In this case, the potential solution $t=-\frac{1}{2}$ need not be explicitly stated.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
8.			•¹ interpret rate of change [1]	•¹ $\frac{dV}{dt} = -0.6$	4
			•² find $\frac{dV}{dh}$ [2]	•² $\frac{dV}{dh} = 18\times6\left(2+h^{\frac{1}{2}}\right)^5 \times \frac{1}{2}h^{-\frac{1}{2}}$	
			•³ form relationship	•³ eg $\frac{dh}{dt} = \frac{dh}{dV}\times\frac{dV}{dt}$	
			•⁴ evaluate $\frac{dh}{dt}$ [3,4,5]	•⁴ $\frac{dh}{dt} = -\frac{1}{93750}\,\text{ms}^{-1}$	

Notes:

1. Where a candidate uses a variable other than t (or T) for time, eg "sec", this must be explicitly defined. Otherwise, the mark where it is first used is not available.
2. Where a candidate equates V to its derivative, the relevant mark is not available.
3. For the award of •⁴, $\frac{dh}{dt} = \ldots$ and units must appear.
4. For the award of •⁴, approximate answers ($-1.0666\ldots\times10^{-5}$) should be rounded to at least 2 significant figures. Accept an equivalent, eg -11×10^{-6} ms^{-1}.
5. For the award of •⁴, there must be a numerical value resulting from a calculation involving at least two substitutions.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
9.			•[1] convert to base 10 [1]	•[1] 497_{10}	3
			•[2] complete method for conversion to base 9	•[2] eg $497 = 9\times 55 + 2$ $55 = 9\times 6 + 1$ $6 = 9\times 0 + 6$	
			•[3] state solution [2]	•[3] 612_9	

Notes:

1. Where a candidate converts 3442_{10} to a number in base 9, •[1] is not available (see COR F).
2. At •[3], disregard the omission of the subscript.

Question		Generic scheme	Illustrative scheme	Max mark
9.	(continued)			

Commonly Observed Responses:

COR A

$497 \div 9 = 55 \text{ r } 2$

$55 \div 9 = 6 \text{ r } 1$

$6 \div 9 = 0 \text{ r } 6$

Award $\bullet^2$

COR B

$497 = 9 \times 55 + 2$

$55 = 9 \times 6 + 1$

leading to 612_9. Award $\bullet^2$ and $\bullet^3$.

COR C

$497 = 9 \times 55 + 2$

$55 = 9 \times 6 + 1$

leading to 216_9. Award $\bullet^2$ but not $\bullet^3$.

COR D

$497 = 9 \times 55 + 2$

$55 = 9 \times 6 + 1$

do not award $\bullet^2$ or $\bullet^3$.

COR E

9^2	9^1	9^0
81	**9**	**1**
6	**1**	**2**

Award $\bullet^2$ for all entries in row 1 and the '6' in row 3. Values of powers of 9 need not be stated.

COR F

$3442 = 9 \times 382 + 4$

$382 = 9 \times 42 + 4$

$42 = 9 \times 4 + 6$

$4 = 9 \times 0 + 4$

leading to 4644. Do not award $\bullet^1$; award $\bullet^2$ and $\bullet^3$.

Question			Generic scheme	Illustrative scheme	Max mark
10.	(a)		•1 evidence of product rule with one term correct	•1 $2xe^{6y}+\dots$ **or** $\dots+x^2 6e^{6y}\dfrac{dy}{dx}$	**4**
			•2 complete differentiation of product	•2 $2xe^{6y}+6x^2e^{6y}\dfrac{dy}{dx}$	
			•3 differentiate remaining terms	•3 $\dots+2x+5y^4\dfrac{dy}{dx}=0$	
			•4 write derivative explicitly in terms of y and x [1,2]	•4 $\dfrac{dy}{dx}=\dfrac{-2xe^{6y}-2x}{5y^4+6x^2e^{6y}}$	

Notes:

1. Where $\dfrac{dy}{dx}$ does not appear more than once after the candidate has completed differentiation, do not award •4.
2. Withhold •4 for incorrect simplification subsequent to a correct answer.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
	(b)		•5 explanation [1,2,3,4,5,6]	•5 eg $e^{6y}\neq-1,\ x\neq0$	**1**

Notes:

1. If the candidate uses an expression which could have a zero numerator, •5 is not available.
2. Do not award •5 where a candidate explicitly considers a zero denominator.
3. Where a candidate uses an incorrect simplification of an expression for which •4 has been withheld, •5 is still available.
4. Do not award •5 if an explanation contains any incorrect statement.
5. Where a candidate factorises the numerator, a statement about x is not required but candidates must state that $e^{6y}\neq-1$, or stronger, eg $e^{6y}>0$.
6. Where a candidate does not factorise the numerator, a statement about x not being 0 is required and candidates must state that $e^{6y}\neq-1$, or stronger, eg $e^{6y}>0$.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
11.			**Method 1**	**Method 1**	4
			•1 decompose integrand [2]	•1 $\frac{2x}{x^2+4}+\frac{4}{x^2+4}$	
			•2 begin to integrate one term [1]	•2 $\ln\left(x^2+4\right)$	
			•3 begin to integrate second term [1]	•3 $\tan^{-1}\left(\frac{x}{2}\right)$	
			•4 complete integration [3]	•4 $\ln\left(x^2+4\right)+2\tan^{-1}\left(\frac{x}{2}\right)+c$	
			Method 2	**Method 2**	
			•1 evidence of integration by parts [2]	•1 $\frac{1}{2}(2x+4)\tan^{-1}\left(\frac{x}{2}\right)-\ldots$	
			•2 complete first application [3]	•2 $\ldots\int\tan^{-1}\left(\frac{x}{2}\right)dx$	
			•3 begin second application	•3 $\ldots\left(x\tan^{-1}\left(\frac{x}{2}\right)-\ldots\right)$	
			•4 complete integration [3]	•4 $\ln\left(x^2+4\right)+2\tan^{-1}\left(\frac{x}{2}\right)+c$	

Notes:

1. In Method 1, at each of •2 and •3, the mark is awarded for the appearance of the given term, regardless of coefficients. These need to be correct for the award of the final mark.
2. Where a candidate adopts any strategy other than exemplified in Methods 1 and 2, the maximum mark available is 1/4, awarded for the appearance of $\ln\left(x^2+4\right)$ or $\tan^{-1}\left(\frac{x}{2}\right)$.
3. Disregard the omission of dx and the absence of the constant of integration.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
12.			•¹ show true for $n=1$ [1,2,3] •² assume true for $n=k$ [3,4,5,6,7] **and** consider whether true for $n=k+1$ [2,3] •³ write $7^{k+1}+2$ in terms of 7^k and 2 •⁴ use inductive hypothesis [8] •⁵ write expression with factor of 3 and communicate [8,9]	•¹ $7^1+2=9$ true for $n=1$ •² suitable statement **and** 7^k+2 is divisible by 3 **or** $7^k+2=3m$, where $m\in\mathbb{Z}$. **and** $7^{k+1}+2=\ldots7^k\ldots$ •³ $7\times7^k+2$ •⁴ $7(3m-2)+2$ •⁵ $3(7m-4)$ **and** If true for $n=k$ then true for $n=k+1$. But true for $n=1$. Therefore true for all $n\in\mathbb{N}$ by induction.	5

Question		Generic scheme	Illustrative scheme	Max mark
12.	(continued)			

Notes:

1. Accept eg "so true" if the candidate has begun with eg "$n=1$:". The statement for $\bullet^1$ may appear at $\bullet^5$.
2. For the award of $\bullet^1$, full substitution (ie 7^1) must appear.
3. Where a candidate uses divisibility notation incorrectly (eg 9|3), withhold the mark where it first appears, but not subsequently. This applies even if the candidate states or implies its meaning.
4. For $\bullet^2$ sufficient phrases for $n=k$ contain:
   - 'If true for…', 'Suppose true for…', 'Assume true for…'.

   For $\bullet^2$, insufficient phrases for $n=k$ contain:
   - 'Consider $n=k$', 'Assume $n=k$', 'Let $n=k$', 'Assume $n=k$ is true', 'True for $n=k$'.

   A sufficient phrase for the award of $\bullet^2$ may appear at $\bullet^5$.

   For $\bullet^2$, unacceptable phrases for $n=k$ contain:
   - 'Assume true $\forall k \in \mathbb{N}$'

   For $\bullet^2$, unacceptable phrases for $n=k+1$ contain:
   - 'Consider true for $n=k+1$', 'True for $n=k+1$'.
5. Do not award $\bullet^2$ for '$7^k+2=3k$' or '$7^k+2=3n$'.
6. For the award of $\bullet^2$, the RHS of the expression for $7^{k+1}+2=\ldots$ must be an expression containing 7^k, or it must be stated that there is a requirement to prove that it is equal to 3 times a letter which is not used elsewhere, or that $7^{k+1}+2$ is divisible by 3.
7. Do not withhold $\bullet^2$ for the omission of a source set.
8. For candidates who have not been awarded $\bullet^4$, $\bullet^5$ is unavailable.
9. Following the required algebra and statement of the inductive hypothesis, the minimal acceptable communication for $\bullet^5$ is "Then true for $n=k+1$. But since true for $n=1$, then true for all n", or equivalent.

Question		Generic scheme	Illustrative scheme	Max mark
12.	(continued)			

Commonly Observed Responses:

COR A (Express $7^{k+1}+2$ as $(6+1)7^k+2$)

•3 $7\times7^k+2$ or $(6+1)7^k+2$

•4 $6\times7^k+3m$

•5 $3\left(2\times7^k+m\right)$ **and** communication.

COR B (Express $7^{k+1}+2$ as $7\left(7^k+2\right)-14+2$)

•3 $7\times7^k+2$ or $7\left(7^k+2\right)-14+2$

•4 $7\times3m-14+2$

•5 $3(7m-4)$ **and** communication.

COR C (Candidate does not write $7^k+2=3m$ but has, instead, stated that this is a multiple of 3).

eg

•3 $7\times7^k+2$

•4 $6\times7^k+7^k+2$

•5 7^k+2 is divisible by 3
and 6×7^k is divisible by 3
and so $7^{k+1}+2$ is divisible by 3
and communication.

COR D (repeated use of letter indicating divisibility in inductive hypothesis)

$$7^1+2=9$$
$$=3m$$
$$7^k+2=3m$$

Do not withhold •2.

Question			Generic scheme	Illustrative scheme	Max mark
13.	(a)		•¹ write template [1] •² find constants and express in partial fractions [1,2,3]	•¹ $\dfrac{x+3}{(x+7)(x+5)}=\dfrac{A}{x+7}+\dfrac{B}{x+5}$ •² $\dfrac{2}{x+7}-\dfrac{1}{x+5}$	2

Notes:

1. For a correct answer without working, award full marks.
2. Accept $\dfrac{2}{x+7}+-\dfrac{1}{x+5}$ or $\dfrac{2}{x+7}+\dfrac{-1}{x+5}$.
3. Where a candidate has written a template, •² may be awarded for $A=2,\ B=-1$.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
13.	(b)		•3 state form of integrating factor [1,3]	•3 $e^{\int \frac{2}{x+3}dx}$	7
			•4 find integrating factor [2,4]	•4 $(x+3)^2$	
			•5 write as integral equation [3,5,6]	•5 $(x+3)^2 y = \int \frac{(x+3)^2}{(x+7)(x+5)(x+3)}dx$	
			•6 rewrite right-hand side using partial fractions [3]	•6 $\int \left(\frac{2}{x+7} - \frac{1}{x+5}\right)dx$	
			•7 integrate [4,7,8,9]	•7 $2\ln(x+7) - \ln(x+5) + c$	
			•8 find constant of integration [7,8,9,10,11]	•8 $-\ln\frac{25}{2}$ **or** $\ln\frac{2}{25}$	
			•9 state particular solution [7,8,9,12]	•9 $y = \dfrac{2\ln(x+7) - \ln(x+5) - \ln\frac{25}{2}}{(x+3)^2}$	

Notes:

1. Where there is no attempt to produce an integrating factor, award 0/7.
2. The simplified version of the integrating factor must not follow from incorrect working.
3. Do not withhold •3, •5 or •6 for the omission of 'dx' on the right-hand side.
4. At •4 and •7, the integration must be of Advanced Higher level.
5. Since a candidate may proceed directly from •4 to •5, disregard incorrect working on the LHS leading to the equation at •5. This equation need not all appear on the same line.
6. Where a candidate produces an incorrect integrating factor, •5 is still available.
7. If, after •6, there is no attempt to integrate, •7, •8 and •9 are not available.
8. Do not withhold •7 for the omission of the constant of integration, but •8 and •9 are unavailable.
9. Where, at •7, a candidate fails to integrate correctly, •8 and •9 are still available.
10. Where a candidate rearranges incorrectly before evaluating the constant of integration, •8 is still available.
11. At •8, accept a decimal approximation (rounded or truncated) to at least 2 significant figures.
12. Incorrect working subsequent to the award of •9 should be ignored.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
14.	(a)		•¹ state two vectors	•¹ any two of eg $\begin{pmatrix}1\\2\\5\end{pmatrix}, \begin{pmatrix}4\\-3\\-2\end{pmatrix}, \begin{pmatrix}3\\-5\\-7\end{pmatrix}$	4
			•² evidence of strategy for finding normal	•² eg $\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\1 & 2 & 5\\4 & -3 & -2\end{vmatrix}$	
			•³ calculate normal [1,2]	•³ $\begin{pmatrix}11\\22\\-11\end{pmatrix}$	
			•⁴ obtain equation [2,3]	•⁴ $x+2y-z=12$	

Notes:

1. For •³, accept any multiple of $\begin{pmatrix}1\\2\\-1\end{pmatrix}$.
2. Where a candidate writes eg $\begin{pmatrix}11\\22\\-11\end{pmatrix}=\begin{pmatrix}1\\2\\-1\end{pmatrix}$, award •³; •⁴ is still available.
3. At •⁴, accept any equivalent equation, eg $11x+22y-11z=132$.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
	(b)		•⁵ find parametric equations of L	•⁵ $x=2t+8$ $y=-t-2$ $z=3t-2$	3
			•⁶ substitute into plane equation	•⁶ $2t+8+2(-t-2)-(3t-2)=12$	
			•⁷ determine coordinates of S [1,2]	•⁷ $(4,0,-8)$	

Notes:

1. Do not accept a point given as a column vector but accept coordinates stated individually.
2. Where, in (a), a candidate has produced the equation of a plane with which the line does not intersect, •⁷ is not available.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
14.	(c)		•8 determine direction vector	•8 $\begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}$, stated **or** implied	3
			•9 begin to calculate angle	•9 $\frac{-3}{\sqrt{14}\sqrt{6}}$ **or** $\frac{3}{\sqrt{14}\sqrt{6}}$	
			•10 calculate angle [1,2,3]	•10 19.1° **or** 0.333 radians	

Notes:

1. At •10, accept an answer rounded or truncated to at least 2 significant figures.
2. At •10, disregard the omission of the degree sign and poor notation, eg equating an angle to its cosine.
3. Do not award •10 where a candidate produces incorrect working following a correct answer.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
15.	(a)		•¹ write down contrapositive [1,2]	•¹ If $\sqrt{r}$ is rational then r is rational	**1**

Notes:

1. Accept "not irrational" in place of "rational".
2. Accept equivalent variations, eg "r is rational if $\sqrt{r}$ is rational" or "$\sqrt{r}$ rational $\Rightarrow r$ rational".

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
	(b)		•² write down appropriate form for $\sqrt{r}$ [1,2,7] •³ state source set and obtain expression for r [3,4,7] •⁴ communicate rationality and state conclusion [4,5,6,7]	•² $\sqrt{r}=\dfrac{p}{q}$, •³ $p,q\in\mathbb{Z}\ (q\neq 0),\ r=\dfrac{p^2}{q^2}$ •⁴ which is rational; contrapositive is true **and** which implies original statement is true	**3**

Notes:

1. At •², the expression must be consistent with the candidate's response to (a). For example, if in (a), a candidate's attempt at writing the contrapositive has an antecedent relating to r, •² is awarded where the candidate proceeds from consideration of the form of r. See CORs C and D.
2. At •², do not accept a fraction which contains the letter r, eg $\sqrt{r}=\dfrac{r}{s}$.
3. Where a candidate's response uses contradiction, •² and •³ are available but •⁴ is not.
4. Do not withhold •³ for the omission of $q\neq 0$.
5. Where a candidate produces a 'contrapositive' which contains 'If r is rational' or 'If r is not rational', •³ and •⁴ are not available.
6. For the award of •⁴, the statement about rationality must relate to a fraction. It must also be consistent with the candidate's response to (a). For example, if in (a), a candidate's attempt at writing the contrapositive ends with a statement about r, •⁴ is awarded where the candidate concludes that r is rational.
7. For the award of •⁴, the conclusion must state that the contrapositive is true and also indicate that this implies the truth of the original statement, eg "so", "$\therefore$" but not "and" or ",".
8. Where a candidate produces an incorrect statement between eg $r=\dfrac{p^2}{q^2}$ and the conclusion, •⁴ is not available.

Question			Generic scheme	Illustrative scheme	Max mark
15.	(b)	(continued)			

Commonly Observed Responses:

COR A

If $\sqrt{r}$ is rational then r is irrational.

$\sqrt{r}=\dfrac{p}{q}$ Award $\bullet^2$.

$p,q\in\mathbb{Z}$, $r=\dfrac{p^2}{q^2}$ Award $\bullet^3$.

Do not award $\bullet^4$.

COR B

If $\sqrt{r}$ is irrational then r is irrational/rational.

$\sqrt{r}\neq\dfrac{p}{q}$ Award $\bullet^2$.

Do not award $\bullet^3$ or $\bullet^4$.

COR C

(following from "r is rational if $\sqrt{r}$ is rational" – here, the antecedent is "$\sqrt{r}$ is rational")

Full marks are available

COR D

(following from "$\sqrt{r}$ is rational if r is rational" – here, the antecedent is "r is rational")

Only $\bullet^2$ is available, and the candidate must begin with "$r=$"

Question			Generic scheme	Illustrative scheme	Max mark
16.	(a)		•¹ begin differentiation •² complete differentiation and communicate	•¹ $\frac{1}{\cos x}\times\ldots$ •² $\frac{1}{\cos x}\times\left(-\sin x\right)=-\tan x$	**2**
Notes:					
Commonly Observed Responses:					
	(b)	(i)	•³ integrate [1] •⁴ complete integration and evaluate [1]	•³ $\ldots\ln\left(\cos 2x\right)$ •⁴ $\frac{\sqrt{3}}{3}\pi+\ln\frac{1}{2}$ **(or** $\frac{\sqrt{3}}{3}\pi-\ln 2$ **)**	**2**
Notes: 1. Where a candidate does not obtain an expression containing $\ln\left(\cos 2x\right)$ at •³, •⁴ is not available.					
Commonly Observed Responses:					
		(ii)	•⁵ state expression	•⁵ $4\sec^2 2x$	**1**
Notes:					
Commonly Observed Responses:					

[END OF MARKING INSTRUCTIONS]