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National
Qualifications
2023

2023 Mathematics

Advanced Higher - Paper 2

Finalised Marking Instructions

General marking principles for Advanced Higher Mathematics

Always apply these general principles. Use them in conjunction with the detailed marking instructions, which identify the key features required in candidates' responses.

For each question, the marking instructions are generally in two sections:

- *generic scheme — this indicates why each mark is awarded*
- *illustrative scheme — this covers methods which are commonly seen throughout the marking*

In general, you should use the illustrative scheme. Only use the generic scheme where a candidate has used a method not covered in the illustrative scheme.

(a) Always use positive marking. This means candidates accumulate marks for the demonstration of relevant skills, knowledge and understanding; marks are not deducted for errors or omissions.

(b) If you are uncertain how to assess a specific candidate response because it is not covered by the general marking principles or the detailed marking instructions, you must seek guidance from your team leader.

(c) One mark is available for each •. There are no half marks.

(d) If a candidate's response contains an error, all working subsequent to this error must still be marked. Only award marks if the level of difficulty in their working is similar to the level of difficulty in the illustrative scheme.

(e) Only award full marks where the solution contains appropriate working. A correct answer with no working receives no mark, unless specifically mentioned in the marking instructions.

(f) Candidates may use any mathematically correct method to answer questions, except in cases where a particular method is specified or excluded.

(g) If an error is trivial, casual or insignificant, for example $6 \times 6 = 12$, candidates lose the opportunity to gain a mark, except for instances such as the second example in point (h) below.

(h) If a candidate makes a transcription error (question paper to script or within script), they lose the opportunity to gain the next process mark, for example

This is a transcription error and so the mark is not awarded.

This is no longer a solution of a quadratic equation, so the mark is not awarded.

$$x^2 + 5x + 7 = 9x + 4$$
$$x - 4x + 3 = 0$$
$$x = 1$$

The following example is an exception to the above

This error is not treated as a transcription error, as the candidate deals with the intended quadratic equation. The candidate has been given the benefit of the doubt and all marks awarded.

$$x^2 + 5x + 7 = 9x + 4$$
$$x - 4x + 3 = 0$$
$$(x-3)(x-1) = 0$$
$$x = 1 \text{ or } 3$$

(i) Horizontal/vertical marking

If a question results in two pairs of solutions, apply the following technique, but only if indicated in the detailed marking instructions for the question.

Example:

	$\bullet^5$	$\bullet^6$
$\bullet^5$	$x=2$	$x=-4$
$\bullet^6$	$y=5$	$y=-7$

Horizontal: $\bullet^5$ $x=2$ and $x=-4$ Vertical: $\bullet^5$ $x=2$ and $y=5$
$\bullet^6$ $y=5$ and $y=-7$ $\bullet^6$ $x=-4$ and $y=-7$

You must choose whichever method benefits the candidate, **not** a combination of both.

(j) In final answers, candidates should simplify numerical values as far as possible unless specifically mentioned in the detailed marking instruction. For example

$\frac{15}{12}$ must be simplified to $\frac{5}{4}$ or $1\frac{1}{4}$ $\frac{43}{1}$ must be simplified to 43

$\frac{15}{0.3}$ must be simplified to 50 $\frac{4/5}{3}$ must be simplified to $\frac{4}{15}$

$\sqrt{64}$ must be simplified to 8*

*The square root of perfect squares up to and including 144 must be known.

(k) Commonly Observed Responses (COR) are shown in the marking instructions to help mark common and/or non-routine solutions. CORs may also be used as a guide when marking similar non-routine candidate responses.

(l) Do not penalise candidates for any of the following, unless specifically mentioned in the detailed marking instructions:

- working subsequent to a correct answer
- correct working in the wrong part of a question
- legitimate variations in numerical answers/algebraic expressions, for example angles in degrees rounded to nearest degree
- omission of units
- bad form (bad form only becomes bad form if subsequent working is correct), for example
  $(x^3+2x^2+3x+2)(2x+1)$ written as
  $(x^3+2x^2+3x+2)\times 2x+1$
  $=2x^4+5x^3+8x^2+7x+2$ gains full credit
- repeated error within a question, but not between questions or papers

(m) In any 'Show that…' question, where candidates have to arrive at a required result, the last mark is not awarded as a follow-through from a previous error, unless specified in the detailed marking instructions.

(n) You must check all working carefully, even where a fundamental misunderstanding is apparent early in a candidate's response. You may still be able to award marks later in the question so you must refer continually to the marking instructions. The appearance of the correct answer does not necessarily indicate that you can award all the available marks to a candidate.

(o) You should mark legible scored-out working that has not been replaced. However, if the scored-out working has been replaced, you must only mark the replacement working.

(p) If candidates make multiple attempts using the same strategy and do not identify their final answer, mark all attempts and award the lowest mark. If candidates try different valid strategies, apply the above rule to attempts within each strategy and then award the highest mark.

For example:

Strategy 1 attempt 1 is worth 3 marks.	Strategy 2 attempt 1 is worth 1 mark.
Strategy 1 attempt 2 is worth 4 marks.	Strategy 2 attempt 2 is worth 5 marks.
From the attempts using strategy 1, the resultant mark would be 3.	From the attempts using strategy 2, the resultant mark would be 1.

In this case, award 3 marks.

Marking Instructions for each question

Question			Generic scheme	Illustrative scheme	Max mark
1.			•[1] start differentiation •[2] apply chain rule [1]	•[1] $\dfrac{2}{\sqrt{1-(3x)^2}}$ •[2] $\dfrac{6}{\sqrt{1-(3x)^2}}$	2

Notes:

1. At •[2] accept $\dfrac{6}{\sqrt{1-9x^2}}$ but do not accept $\dfrac{6}{\sqrt{1-3x^2}}$.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
2.			•[1] evidence of recognising $\displaystyle\int \frac{f'(x)}{f(x)}dx$ [1,2] •[2] determine coefficient of $\ln\lvert x^3+10\rvert$ [1,2,3]	•[1] $k\ln\lvert x^3+10\rvert,\ k\in\mathbb{R}$ •[2] $\dfrac{1}{3}\ln\lvert x^3+10\rvert+c$	2

Notes:

1. •[1] and •[2] are available only for an expression of the form $k\ln\lvert x^3+10\rvert,\ k\in\mathbb{R}$, with no further x terms.
2 Do not withhold •[1,2] for the omission of modulus signs.
3. Do not withhold •[2] for the omission of the constant of integration.

Commonly Observed Response:

Integration by Substitution:

$\dfrac{1}{3}\displaystyle\int\frac{du}{u}$ award •[1]

$\dfrac{1}{3}\ln\lvert x^3+10\rvert+c$ award •[2]

Question			Generic scheme	Illustrative scheme	Max mark
3.	(a)		•[1] begin to find determinant •[2] simplify [1]	•[1] eg $2\begin{vmatrix} -1 & 0 \\ 0 & -2 \end{vmatrix} - 2x\begin{vmatrix} x & 0 \\ 1 & -2 \end{vmatrix} + 4\begin{vmatrix} x & -1 \\ 1 & 0 \end{vmatrix}$ •[2] $4x^2+8$	2

Notes:

1. Do not award •[2] where a candidate incorrectly simplifies $4x^2+8$.

Commonly Observed Response:

Rule of Sarrus

$$\begin{array}{ccccc} 2 & 2x & 4 & 2 & 2x \\ x & -1 & 0 & x & -1 \\ 1 & 0 & -2 & 1 & 0 \end{array}$$

award •[1]

$4x^2+8$ award •[2]

Question			Generic scheme	Illustrative scheme	Max mark
3.	(b)		•³ state conclusion [1,2,3,4]	•³ eg $4x^2+8\neq 0$ A^{-1} always exists	1

Notes:

1. Accept $4x^2+8>0$.
2. Do not withhold •³ for omission of "always".
3. Accept 'yes' as conclusion after $4x^2+8\neq 0$.
4. Where the answer contains incorrect information (before, between or after correct information), •³ is not available.

Commonly Observed Responses:

COR A – Candidate produces a quadratic expression which would have a negative discriminant

$\mathbf{4x^2+2x+8\neq 0}$, so A^{-1} (always) exists — award •³

COR B

$4x^2\neq 0$

$x\neq 0$, so A^{-1} does not always exist — award •³

COR C

$\mathbf{4x^2+8=0}$

$\mathbf{x=(\pm)\sqrt{-2}}$,

so A^{-1} (always) exists — award •³

COR D

$\mathbf{4x^2+8=0}$

$\mathbf{x=(\pm)\sqrt{-2}}$,

so A^{-1} (always) exists except when $\mathbf{x=(\pm)\sqrt{-2}}$ — do not award •³

COR E

$\mathbf{4x^2+8=0}$

$\mathbf{x=(\pm)\sqrt{2}i}$, so A^{-1} does not exist for $x=(\pm)\sqrt{2}i$ — do not award •³

Question			Generic scheme	Illustrative scheme	Max mark
4.			•1 begin differentiation of product term, with one term correct •2 complete differentiation of product term •3 complete differentiation and calculate gradient	•1 $2xy^2+\ldots$ or $\ldots+2x^2y\frac{dy}{dx}$ •2 $2xy^2+2x^2y\frac{dy}{dx}$ •3 $2xy^2+2x^2y\frac{dy}{dx}-2\frac{dy}{dx}=3\cos 3x$ leading to $-\frac{3}{2}$	3

Notes:

1. Where the differentiation of the product term produces one term only, •1 and •2 are not available.
2. At •3, accept $\frac{3}{-2}$.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
5.	(a)		•¹ state general term [1,3] •² simplify powers of x or coefficients [3] •³ state simplified general term [2,3,4]	•¹ $\binom{8}{r}(3x)^{8-r}\left(\frac{-2}{x^2}\right)^r$ •² $3^{8-r}(-2)^r$ or x^{8-3r} •³ $\binom{8}{r}3^{8-r}(-2)^r x^{8-3r}$	3

Notes

1. Candidates may also proceed from $\binom{8}{r}(3x)^r\left(\frac{-2}{x^2}\right)^{8-r}$.
2. Award full marks if the expression at •³ appears without working.
3. Where a candidate writes out a full expansion, •¹, •² and •³ are not available, unless the general term is identifiable in (b).
4. Where a candidate in (a) produces an incorrect further simplification subsequent to the correct answer (eg $3^{8-r}(-2)^r$ becomes $(-6)^8$), •³ is not available.

Commonly Observed Responses:

COR A

General term has not been isolated

$$\sum_{r=0}^{8}\binom{8}{r}(3x)^{8-r}\left(\frac{-2}{x^2}\right)^r$$

$$=\sum_{r=0}^{8}\binom{8}{r}(3)^{8-r}(-2)^r x^{8-3r}$$

Do not award •¹. Award •² and •³.

COR B

General term has been isolated

$$\sum_{r=0}^{8}\binom{8}{r}(3x)^{8-r}\left(\frac{-2}{x^2}\right)^r$$

$$=\binom{8}{r}(3)^{8-r}(-2)^r x^{8-3r}$$

Disregard the incorrect use of the final equals sign. Award •¹, •² and •³.

COR C

Binomial expression has been equated to the general term

$$\left(3x-\frac{2}{x^2}\right)^8=\binom{8}{r}(3x)^{8-r}\left(\frac{-2}{x^2}\right)^r$$

Disregard the incorrect use of the equals sign. Award •¹.

COR D

Negative sign omitted

$$\binom{8}{r}(3x)^{8-r}\left(\frac{2}{x^2}\right)^r$$

Do not award •¹, but •² and •³ are still available.

COR E

Brackets omitted around -2

$$\binom{8}{r}(3)^{8-r}-2^r x^{8-3r}$$

Do not award •³.

Question			Generic scheme	Illustrative scheme	Max mark
5.	(b)		•[4] determine value of r [2] •[5] find coefficient [1,2]	•[4] 3 •[5] -108864	2

Notes:

1. Do not withhold •[5] for an answer of $-\dfrac{108864}{x}$.
2. Where a candidate writes out a full expansion, •[4] may be awarded only if the expansion is complete and correct at least as far as the required term (in either direction). The required term must clearly identified in the expansion for •[5] to be awarded.

Commonly Observed Response:

Binomial expansion

$$6561x^8 - 34992x^5 + 81648x^2 - 108864x^{-1} + 90720x^{-4} - 48384x^{-7} + 16128x^{-10} - 3072x^{-13} + 256x^{-16}$$

Question			Generic scheme	Illustrative scheme	Max mark
6.	(a)		•[1] obtain d [1]	•[1] 19	**1**

Notes:

1. For the award of •[1], 19 must be clearly identified as the gcd in (a) or implied by its use in (b).

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
	(b)		•[2] express gcd in terms of 304 and 399 •[3] find values of a and b [1]	•[2] $19 = 304 - 3 \times 95$ $= 304 - 3 \times (399 - 1 \times 304)$ •[3] $a = 4, b = -7$	**2**

Notes:

1. At •[3], where candidates do not explicitly communicate a and b:
   - accept $19 = 4 \times 703 + (-7) \times 399$ and $19 = 703 \times 4 + 399 \times (-7)$
   - do not accept $19 = 4 \times 703 - 7 \times 399$ or $19 = 4 \times 703 - 399 \times 7$

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
	(c)		•[4] find values of p and q [1,2]	•[4] $p = 16, q = -28$	**1**

Notes:

1. Do not accept $a = 16, b = -28$.
2. At •[4], where candidates do not explicitly communicate p and q:
   - accept $76 = 16 \times 703 + (-28) \times 399$ and $76 = 703 \times 16 + 399 \times (-28)$
   - do not accept $76 = 16 \times 703 - 28 \times 399$ $76 = 16 \times 703 - 399 \times 28$ unless •[3] has already been withheld on the same grounds.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
7.	(a)		•[1] find integrating factor [1] •[2] write as integral equation [2,3] •[3] integrate right-hand side [4,5] •[4] find particular solution [5,6]	•[1] e^{-2x} •[2] $e^{-2x}y=6\int e^{5x}e^{-2x}\,dx$ •[3] $6\times\frac{1}{3}e^{3x}+c$ •[4] $y=2e^{5x}-3e^{2x}$	4

Notes:

1. For any attempt to separate variables, award 0/4.
2. Do not withhold •[2] for the omission of 'dx' on the right-hand side.
3. Where a candidate writes $\int\frac{d}{dx}\ldots$ on the left-hand side do not withhold •[2] provided the candidate later produces evidence that they have integrated with respect to x.
4. Where a candidate integrates $6e^{5x}$, •[3] is not available.
5. For a candidate who omits the constant of integration, •[3] may be available but •[4] is not.
6. Accept $y=\frac{2e^{3x}-3}{e^{-2x}}$ at •[4].

Commonly Observed Response:

Candidate treats equation as linear differential equation:

•[1] write auxiliary equation and obtain complementary function: $m-2=0$ and $y=Ae^{2x}$

•[2] particular integral and derivative: $y=Be^{5x}$ and $\frac{dy}{dx}=5Be^{5x}$

•[3] substitute PI into differential equation and determine B: $5Be^{5x}-2Be^{5x}=6e^{5x}$ and $B=2$

•[4] find particular solution: $y=2e^{5x}-3e^{2x}$

Question			Generic scheme	Illustrative scheme	Max mark
	(b)		•[5] find third derivative [1,2] •[6] find k [2,3]	•[5] $\frac{d^3y}{dx^3}=250e^{5x}-24e^{2x}$ •[6] k = 36	2

Notes:

1. Where a candidate recognises from the RHS that the e^{5x} term need not be considered, •[5] is available only where this is clearly stated.
2. Where a candidate adopts an approach using an auxiliary equation, •[5] is available for a general complementary function of the form $y=A+Bx+Ce^{5x}$ AND a particular integral of the form $y=De^{2x}$. For the award of •[6], candidates must derive the value of k from their coefficient of e^{2x} in (a).
3. Accept $36e^{2x}$ for the award of •[6].

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
8.	(a)	(i)	•¹ find the common ratio	•¹ 3	**1**

Notes:

Commonly Observed Responses:

		(ii)	•² find first term	•² $\frac{1}{3}$	**1**

Notes:

Commonly Observed Responses:

	(b)		•³ find S_n and S_{2n} [1] •⁴ obtain expression [2]	•³ $S_n = \frac{\frac{1}{3}\left(1-3^n\right)}{1-3}$ and $S_{2n} = \frac{\frac{1}{3}\left(1-3^{2n}\right)}{1-3}$ •⁴ $\frac{\left(1-3^n\right)\left(1+3^n\right)}{1-3^n}$ leading to $1+3^n$	**2**

Notes:

1. For the award of •³, where a candidate produces a fractional common ratio in (a), brackets must be used when substituting, unless they separately consider numerator and denominator.
2. For the award of •⁴, there must be evidence that the candidate has completed a difference of two squares, or multiplied numerator and denominator by $1+3^n$.

Commonly Observed Response:

For candidates who deal with general expressions:

$\frac{1-r^{2n}}{1-r^n}$ or $\frac{r^{2n}-1}{r^n-1}$ award •³

Question			Generic scheme	Illustrative scheme	Max mark
9.				$572 = 9 \times 63 + 5$	2
			•¹ evidence of valid method ¹	•¹ $63 = 9 \times 7 + 0$ $7 = 9 \times 0 + 7$	
			•² express in base nine ²,³	•² 705_9	

Notes:

1. For the award of •¹ , the final expression must contain a zero quotient.
2. For the award of •², at least three digits must be produced from a valid method.
3. At •², disregard omission of base 9.

Commonly Observed Responses:

COR A

$572 \div 9 = 63 \text{ r } 5$

$63 \div 9 = 7 \text{ r } 0$

$7 \div 9 = 0 \text{ r } 7$

leading to 705 award •¹ and •²

COR B

$572 = 9 \times 63 + 5$

$63 = 9 \times 7 + 0$

leading to 705 award •¹ and •²

COR C

$572 = 9 \times 63 + 5$

$63 = 9 \times 7 + 0$

leading to 507 award •¹ but not •²

COR D

9^2	9^1	9^0
81	9	1
7	0	5

Award •¹ for all entries in row 2 and the ‘7’ in row 3.

Question			Generic scheme	Illustrative scheme	Max mark
10.			•1 take logarithms on both sides and apply rule [1]	•1 $\ln y = 5x^2 \ln x$	5
			•2 differentiate $\ln y$	•2 $\frac{1}{y}\frac{dy}{dx}$	
			•3 evidence of product rule with one term correct [2,3]	•3 $10x\ln x + \ldots$ or $\ldots + 5x^2 \cdot \frac{1}{x}$	
			•4 complete differentiation [2,3]	•4 $10x\ln x + 5x^2 \cdot \frac{1}{x}$	
			•5 write $\frac{dy}{dx}$ in terms of x [1,3,4]	•5 $\frac{dy}{dx} = x^{5x^2}\left(10x\ln x + 5x\right)$	

Notes:

1. Accept 'log' as an alternative to '$\ln$' provided candidate does not indicate a base other than e. For candidates who do use a base other than e, only •1 and •5 are available.
2. For candidates who do not attempt to use the product rule, •3 and •4 are not available.
3. Accept '$5x$' instead of '$5x^2 \cdot \frac{1}{x}$' for •3 and •4 . However, do not accept '$5x^2 \cdot \frac{1}{x}$' for •5 .
4. •5 is not available for candidates who subsequently produce an incorrect statement - eg $x^{5x^2}\left(10x\ln x + 5x\right)$ becomes $x^{6x^2}\left(10\ln x + 5\right)$.

Commonly Observed Responses:

COR A

For candidates who write $y = e^{\ln x^{5x^2}}$, marks may be awarded as follows.

•1 write in the form $y = e^{\ln x^{5x^2}}$.

•2,3 apply chain rule $\frac{dy}{dx} = e^{5x^2 \ln x} \cdot \frac{d}{dx}\left(5x^2 \ln x\right)$

•4 use product rule with one term correct $10x\ln x + \ldots$ or $\ldots + 5x^2 \cdot \frac{1}{x}$

•5 complete differentiation $\frac{dy}{dx} = x^{5x^2}\left(10x\ln x + 5x\right)$ or $\frac{dy}{dx} = e^{5x^2 \ln x}\left(10x\ln x + 5x\right)$

COR B

For candidates who write $y = e^{5x^2}\ln x$ do not award •1, •2 or •5. However, •3 and •4 are still available for a correct application of the product rule.

Question			Generic scheme	Illustrative scheme	Max mark
11.	(a)		•¹ determine the relationship between r and h	•¹ $r=\frac{90h}{150}$ (or equivalent) leading to $V=\frac{3\pi h^3}{25}$	1

Notes:

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
	(b)		•² find $\frac{dV}{dh}$ ²	•² $\frac{dV}{dh}=\frac{9\pi h^2}{25}$	5
			•³ form relationship ³	•³ eg $\frac{dV}{dt}=\frac{dV}{dh}\frac{dh}{dt}$ stated or implied at •⁵	
			•⁴ interpret rate of change of V in cm^3s^{-1}	•⁴ $\frac{dV}{dt}=10000$	
			•⁵ form expression for $\frac{dh}{dt}$ in terms of h ⁵	•⁵ $\frac{dh}{dt}=\frac{25\times 10000}{9\pi h^2}$	
			•⁶ evaluate $\frac{dh}{dt}$ 1,4,6	•⁶ $\frac{16}{9\pi}\text{ cms}^{-1}$	

Notes:

1. Where alternative notation is used for V, h and t, •⁶ is available only if the variables are defined and used consistently.
2. If a derivative is equated to the original expression, •² is not available.
3. At •³ accept any correct form of the chain rule which involves both $\frac{dh}{dt}$ and $\frac{dV}{dt}$.
4. Accept an answer rounded to at least 2 significant figures $\left(0.57\text{ cms}^{-1}\right)$.
5. For the award of •⁵ the expression need not be simplified.
6. Units are required for •⁶.

Commonly Observed Response:

Where a candidate does not convert litres to cm³.

$\frac{dV}{dt}=10$ do not award •⁴

•⁵ $\frac{dh}{dt}=\frac{25\times 10}{9\pi h^2}$

•⁶ $\frac{2}{1125\pi}\text{cms}^{-1}$ or $5.7\times 10^{-4}\text{cms}^{-1}$

Question			Generic scheme	Illustrative scheme	Max mark
12.			●[1] show true for $n = 1$ [1]	●[1] $(\text{LHS} =) 2^{1-1} \times 1 = 1$ $(\text{RHS} =) 2^1(1-1)+1 = 1$ so result is true when $n = 1$	5
			●[2] assume statement true for $n = k$ AND consider whether statement true for $n = k+1$ [2,5]	●[2] suitable statement AND $\sum_{r=1}^{k} 2^{r-1} r = 2^k(k-1)+1$ AND $\sum_{r=1}^{k+1} 2^{r-1} r = \cdots$	
			●[3] state sum to $k+1$ terms using inductive hypothesis [3]	●[3] $2^k(k-1)+1+(k+1)2^{k+1-1}$ or $2^k(k-1)+1+(k+1)2^k$	
			●[4] take out common factor of 2^k and simplify [4]	●[4] $2^k.2k+1$	
			●[5] express sum explicitly in terms of $(k+1)$ or achieve stated aim/goal AND communicate [5,6,7]	●[5] $2^{(k+1)}(k+1-1)+1$ **AND** If true for $n = k$ then true for $n = k+1$. Also true for $n = 1$ therefore, by induction, true for all positive integers n	

Question		Generic scheme	Illustrative scheme	Max mark
12.	(continued)			

Notes:

1. "RHS = 1, LHS = 1" and/or "True for $n = 1$" are insufficient for the award of $\bullet^1$. Where a candidate does not independently evaluate LHS and RHS, $\bullet^1$ may still be awarded.

2. For $\bullet^2$, sufficient phrases for $n = k$ contain:
   - 'If true for...'; 'Suppose true for…'; 'Assume true for…'.

   For $\bullet^2$ insufficient phrases for $n = k$ contain:
   - 'Consider $n = k$', 'assume $n = k$', 'assume $n = k$ is true' and 'True for $n = k$'.

   A sufficient phrase for the award of $\bullet^2$ may appear at $\bullet^5$.

   For $\bullet^2$, accept:

   assume true for $n = k$ AND $\sum_{r=1}^{k} 2^{r-1} r = 2^k (k-1) + 1$ AND "Aim/goal: $\sum_{r=1}^{k+1} 2^{r-1} r = 2^{k+1}(k+1-1)+1$"

   For $\bullet^2$ unacceptable phrases for $n = k+1$ contain:
   - "Consider true for $n = k+1$", "true for $n = k+1$",
   - "$\sum_{r=1}^{k+1} 2^{r-1} r = 2^{k+1}(k+1-1)+1$" (with no reference to aim/goal and no further processing).

3. Since $\bullet^3$ may be awarded directly after $\bullet^2$, exercise leniency in relation to intermediate notation where this has been provided, eg the handling of sigma notation.

4. $\bullet^4$ is unavailable to candidates who arrive at $2^k.2k+1$ without algebraic justification.

5. $\bullet^5$ is unavailable to candidates who have not been awarded $\bullet^4$.

6. Full marks are available to candidates who state an aim/goal earlier in the proof and who subsequently achieve the stated aim/goal, provided $2^{k+1}(k+1-1)+1$ appears at some point.

7. Following the required algebra and statement of the inductive hypothesis, the minimal acceptable response for $\bullet^5$ is: "Then true for $n = k+1$, but since true for $n = 1$, then true for all n" or equivalent.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
13.			•[1] write as integral equation [1] •[2] integrate P expression •[3] integrate m expression [2] •[4] substitute values following integration [2] •[5] evaluate constant of integration [2,4] •[6] write expression in terms of m [2,3,4]	•[1] $\int \frac{1}{P} dP = \int \frac{1 \cdot 4}{m-220} dm$ •[2] $\ln P$ •[3] $1 \cdot 4 \ln(m-220)+c$ •[4] $\ln 1079 = 1 \cdot 4 \ln(807-220)+c$ •[5] $-1 \cdot 94$ •[6] $P = 0 \cdot 14(m-220)^{1.4}$	6

Notes:

1. Do not award •[1] where $\int \ldots dP$ and $\int \ldots dm$ do not appear.
2. For candidates who omit the constant of integration, •[3] may be awarded but •[4], •[5] and •[6] are unavailable.
3. For •[6] accept $P = e^{1 \cdot 4 \ln(m-220) - 1 \cdot 94}$ or equivalent.
4. Disregard numerical errors due to truncation or premature rounding.

Question	Generic scheme	Illustrative scheme	Max mark

13. (continued)

Commonly Observed Responses:

Alternative Correct Solutions

COR A (rearranging before substitution)

•4 $P=e^{1.4\ln(m-220)+c}$

•5 $1079=e^{1.4\ln(807-220)}e^{c}$

•6 $P=0\cdot 14(m-220)^{1.4}$

COR B

•1 $\int \frac{1}{1\cdot 4P}dP=\int \frac{1}{m-220}dm$

•2 $\frac{1}{1.4}\ln P$

•3 $\ln(m-220)+c$

•4 $\frac{1}{1\cdot 4}\ln 1079=\ln(807-220)+c$

•5 $-1\cdot 39$

•6 $P=0\cdot 14(m-220)^{1.4}$

COR C

•1 $\int \frac{1}{1\cdot 4P}dP=\int \frac{1}{m-220}dm$

•2 $\frac{1}{1.4}\ln 1.4P$

•3 $\ln(m-220)+c$

•4 $\frac{1}{1\cdot 4}\ln(1.4\times 1079)=\ln(807-220)+c$

•5 $-1\cdot 15$

•6 $P=0\cdot 14(m-220)^{1.4}$

Incorrect Integration of LHS

COR D

•1 $\int \frac{1}{1\cdot 4P}dP=\int \frac{1}{m-220}dm$

$1.4\ln P$ do not award •2

•3 $\ln(m-220)+c$

•4 $1\cdot 4\ln 1079=\ln(807-220)+c$

•5 3.40

•6 $P=11.3(m-220)^{0.71}$

COR E

•1 $\int \frac{1}{1\cdot 4P}dP=\int \frac{1}{m-220}dm$

$1.4\ln 1.4P$ do not award •2

•3 $\ln(m-220)+c$

•4 $1\cdot 4\ln(1.4\times 1079)=\ln(807-220)+c$

•5 3.87

•6 $P=11.3(m-220)^{0.71}$

COR F

•1 $\int \frac{1}{1\cdot 4P}dP=\int \frac{1}{m-220}dm$

$\ln 1.4P$ do not award •2

•3 $\ln(m-220)+c$

•4 $\ln(1.4\times 1079)=\ln(807-220)+c$

•5 0.945

$P=1.84(m-220)$ do not award •6 (eased)

Question	Generic scheme	Illustrative scheme	Max mark
13. (continued)			
		<u>Incorrect Integration of LHS</u> **COR G** •¹ $\int \frac{1}{1\cdot 4P}dP = \int \frac{1}{m-220}dm$ $\ln 1.4P$ — do not award •² •³ $\ln(m-220)+c$ $P=\frac{(m-220)e^{c}}{1.4}$ — do not award •⁴ (eased) •⁵ $1079=\frac{(807-220)e^{c}}{1.4}$ •⁶ $P=1.84(m-220)$	

Question			Generic scheme	Illustrative scheme	Max mark
14.			•¹ substitute, expand and apply $i^2=-1$ [1]	•¹ a^2-b^2+2abi	4
			•² equate real and imaginary parts	•² $a^2-b^2=8$ and $2ab=6$	
			•³ substitute for b or a [2]	•³ eg $a^2-\frac{9}{a^2}=8$	
			•⁴ rearrange into quartic in standard form and solve [2,3,4]	•⁴ $a^4-8a^2-9=0$ and $a=3,\ b=1$	

Notes:

1. The imaginary part need not be simplified for the award of •¹.
2. For the award of •³ and •⁴, there must be suitable algebraic justification. Answers obtained by trial and error are not acceptable.
3. For the award of •⁴, a and b must both be positive.
4. Provided appropriate algebraic justification is present, •⁴ may be awarded for $w=3+i$.

Commonly Observed Responses:

Candidates who use de Moivre's theorem:

•¹ determine modulus or argument	$\lvert w^2\rvert=10$ or $\theta=36.9^\circ$ (0.64 radians)
•² express in polar form	$10\left(\cos 36.9^\circ+i\sin 36.9^\circ\right)$ or $10\left(\cos 0.64+i\sin 0.64\right)$
•³ apply de Moivre's theorem	$\sqrt{10}\left(\cos\frac{36.9^\circ}{2}+i\sin\frac{36.9^\circ}{2}\right)$ or $\sqrt{10}\left(\cos\frac{0.64}{2}+i\sin\frac{0.64}{2}\right)$

(Do not award •³ if the argument of w^2 is 0 or of the form $2k\pi$ or $360k^\circ$, $k\in\mathbb{Z}$.)

•⁴ determine a and b	$a=3,\ b=1$ or $a=3,\ b=0.99$

Question			Generic scheme	Illustrative scheme	Max mark
15.	(a)		•[1] evidence of use of quotient rule with denominator and one term of numerator correct [1]	•[1] $\dfrac{1+(x+1)^4\ldots}{\left(1+(x+1)^4\right)^2}$ **OR** $\dfrac{\ldots-(x+1)4(x+1)^3}{\left(1+(x+1)^4\right)^2}$	**3**
			•[2] complete differentiation	•[2] $\dfrac{1+(x+1)^4-(x+1)4(x+1)^3}{\left(1+(x+1)^4\right)^2}$	
			•[3] find required terms	•[3] $1+\frac{1}{2}x-\frac{1}{4}x^2$	

Notes:

1. Where a candidate arrives at $1+\frac{1}{2}x-\frac{1}{4}x^2$ without algebraic differentiation, award •[3] only.

Commonly Observed Responses:

COR A
Alternative Method (Product Rule)

•[1] $\left(1+(x+1)^4\right)\ldots$

•[2] $\ldots+(x+1)(-1)\left(1+(x+1)^4\right)^{-2}4(x+1)^3$

COR B
Candidates who use logarithmic differentiation

•[1] $\ln\left(f'(x)\right)=\ln(x+1)-\ln\left(1+(x+1)^4\right)$

•[2] $\dfrac{1}{f'(x)}f''(x)=\dfrac{1}{x+1}-\dfrac{4(x+1)^3}{1+(x+1)^4}$ (candidates may be less precise on LHS)

Question			Generic scheme	Illustrative scheme	Max mark
15.	(b)		•4 find $\frac{du}{dx}$ 1 •5 rewrite integral in terms of u 1 •6 integrate and substitute for u 2	•4 $\frac{du}{dx}=2(x+1)$ •5 $\frac{1}{2}\int\frac{du}{1+u^2}$ •6 $\frac{1}{2}\tan^{-1}(x+1)^2+c$	**3**
Notes: 1. $\frac{du}{dx}$ may be implied at •5. 2. Do not withhold •6 for the omission of the constant of integration.					
Commonly Observed Responses:					
	(c)		•7 interpret Maclaurin expansion 1 •8 obtain $f(x)$ 2	•7 $f(0)=1$ •8 $\frac{1}{2}\tan^{-1}(x+1)^2+1-\frac{\pi}{8}$	**2**
Notes: 1. At •7 accept $\frac{1}{2}\tan^{-1}(0+1)^2+c=1$. 2. Award •8 for $c=1-\frac{\pi}{8}$ provided candidate has previously written $\frac{1}{2}\tan^{-1}(x+1)^2+c$.					
Commonly Observed Responses:					

[END OF MARKING INSTRUCTIONS]